Mathematical Programming

Part-A

a) Define supporting hyperplane.

b) Write the difference between stage and state in Dynamic programming problem.

c) Write the mathematical formulations of linear fractional programming problem.

d) Define integer linear programming problem.

e) Define additively separable function.

f) Show that every hyperplane is a convex set.

g) Define pure integer programming problem.

h) Write the constraint system for standard form I in revised simplex method.

i) Which type of models can be solved using goal programming.

j) What do you mean by absolute goal.

k) Write the condition when a L.P.P has unbounded feasible region but bounded optimal solution

l) To the constraint equation $\frac{12}{7} = x_2 - \frac{1}{7} s_1 + \frac{5}{14} s_2$ write corresponding Gomory's constraint.

m) State mixed integer programming problem.

define a basic ~~solution~~ solution.

q) What are advantages of revised simplex method over the original simplex method

r) Explain the cutting plane method of solving an integer problem.

s) What do you mean by cardinal value and ordinal value in the goal programming

t) When do you terminate the branch and bound procedure for a maximisation problem.

Part - B

(1) Solve the following LPP by the revised Simplex Method

$$\max z = x_1 + x_2$$

s.t. $3x_1 + 2x_2 \leq 6$

$x_1 + 4x_2 \leq 4$

and $x_1, x_2 \geq 0$.

(2) Solve the following LPP by bounded variable technique:

$$\text{maximize } z = 3x_1 + 5x_2 + 2x_3$$

s.t. $x_1 + 2x_2 + 2x_3 \leq 14$

$2x_1 + 4x_2 + 3x_3 \leq 23$

and $0 \leq x_1 \leq 4$, $2 \leq x_2 \leq 5$, $0 \leq x_3 \leq 3$.

(3) Solve the following by branch and bound technique.

$$\max z = x_1 + x_2.$$

s.t $3x_1 + x_2 \leq 12$

$x_2 \leq 2$

and $x_1, x_2 \geq 0$ and both are integers.

(4) A company produces two products A and B which require 3 and 4 hours of plant capacity per unit. Total plant capacity is 50 hours/week. The unit profit of products A and B are ₹100 and ₹70 respectively. The company's only goal is to earn a profit of exactly ₹1000 per week. Formulate this problem as a G.P.P. and solve it

(5) Using Gomory's cutting plane method, solve the following LPP.

$$\max z = x_1 + x_2$$

$$\text{s.t. } 2x_1 + 5x_2 \leq 16$$

$$6x_1 + 5x_2 \leq 30$$

$x_1, x_2 \geq 0$ and x_1 and x_2 are integers.

(6) Use separable programming algorithm to solve the non-linear programming problem:

$$\max z = x_1 + x_2^4$$

$$\text{s.t. } 3x_1 + 2x_2^2 \leq 9$$

and $x_1, x_2 \geq 0$

(7) Solve the following linear fractional programming problem:

$$\max z = \frac{-3x_1 - x_2}{x_1 + 2x_2 + 5}$$

Subject to $x_1 + x_2 \geq 1$

$$2x_1 + 3x_2 \geq 2$$

$$x_1, x_2 \geq 0.$$

(8) Define separable convex programming problem and prove that the set of all optimal solution of the convex programming is a convex set

(9) Use Bellman's principle of optimality to minimize: $Z = x_1 + x_2 + \dots + x_n$

$$\text{s.t. } x_1 \cdot x_2 \cdot \dots x_n = d$$

and $x_i \geq 0$ for $i = 1, 2, \dots, n$.

(10) Solve the following linear programming problem by dynamic programming approach:

$$\max z = 2x_1 + 5x_2$$
$$\text{s.t. } 2x_1 + x_2 \leq 43$$
$$2x_2 \leq 46$$
$$\text{and } x_1, x_2 \geq 0$$

(11) Solve following LPP by using bounded variable technique: $\text{Max } z: x_2 + 3x_3$

$$\text{s.t. } x_1 + x_2 + x_3 \leq 10$$
$$x_1 - 2x_3 \geq 0$$
$$2x_2 - x_3 \leq 10$$
$$\text{and } 0 \leq x_1 \leq 8, \quad 0 \leq x_2 \leq 4, \quad x_3 \geq 0.$$

(12) Solve the following LPP by the revised simplex method: $\text{Max } z = 5x_1 + 3x_2$

$$\text{s.t. } 4x_1 + 5x_2 \geq 10$$
$$5x_1 + 2x_2 \leq 10$$
$$3x_1 + 8x_2 \leq 12 \quad \text{and } x_1, x_2 \geq 0$$

(13) Using bounded variable simplex method, solve the following LPP - $\text{Max } z = 3x_1 + x_2 + x_3 + 7x_4$

$$\text{S.t. } 2x_1 + 3x_2 - x_3 + 4x_4 \leq 40$$
$$-2x_1 + 2x_2 + 5x_3 - x_4 \leq 35$$
$$x_1 + x_2 - 2x_3 + 3x_4 \leq 100$$
$$\text{and } x_1 \geq 2, x_2 \geq 1, x_3 \geq 3, x_4 \geq 4.$$

(14) Solve the following fractional programming problem:

Maximize $z = \frac{5x_1 + 3x_2}{5x_1 + 2x_2 + 1}$

subject to $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

and $x_1, x_2 \geq 0$.

(15) Using dynamic programming to solve -

Maximize $z = 2x_1 + 5x_2$

subject to $2x_1 + x_2 \leq 43$

$x_2 \leq 23$ and $x_1, x_2 \geq 0$

(16) Let $X \subset R^n$ be a closed convex set. Prove that for any point y not in the set X, there is a hyperplane containing y such that X is contained in one of the open half spaces determined by the hyperplane.

(17) Solve following LPP by Gromory's method:

Max $z = 7x_1 + 9x_2$

s.t. $-x_1 + 3x_2 \leq 6$

$7x_1 + x_2 \leq 35$

and $x_1, x_2 \geq 0$ and are integers.

(18) Solve the following separable programming problem

Minimize: $z : x_1^2 - 2x_1 - x_2$

such that $2x_1^2 + 3x_2^2 \leq 6$

And x_1

(19) Solve the following goal programming problem by using graphical method:

Minimize $Z = P_1 d_1^- + 2P_2 d_2^- + P_2 d_3^- + P_3 d_1^+$

subject to, $x_1 + x_2 + d_1^- - d_1^+ = 100$

$x_1 + d_2^- = 60$

$x_2 + d_3^- = 80$

and $x_1, x_2, d_1^-, d_1^+, d_2^-, d_3^- \geq 0$

(20) Use dynamic programming to solve:

Maximize $Z = 2x_1 + 5x_2$

Subject to $2x_1 + x_2 \leq 43$

$x_2 \leq 23$

and $x_1, x_2 \geq 0$.

(21) Explain the concept of dynamic programming. Show how to solve a linear programming problem by dynamic programming techniques.

(22) Prove that every supporting hyperplane of a closed convex set which is bounded from below contains at least one extreme point of set.

(23) Find optimum integer solution by Gomory's method to LPP:

Max $Z = x_1 + 2x_2$

such that $2x_2 \leq 7$

$x_1 + x_2 \leq 7$

$2x_1 \leq 11$ and $x_1, x_2 \geq 0$ are integers

(24) Solve the following LPP by branch and bounds technique. Max $z = x_1 + x_2$

S.t. $3x_1 + 2x_2 \leq 12$.

$x_2 \leq 2$

and $x_1, x_2 \geq 0$ are integers

(25) Solve the goal programming problem by using graphical method -

Minimize. $z = p_1 d_1^- + 2p_2 d_2^- + 3p_2 d_3^-$

S.t. $x_1 + x_2 + d_1^- = 80$

$x_1 + d_2^- = 60$

$x_2 + d_3^- = 50$

and $x_1, x_2, d_1^-, d_2^-, d_3^- \geq 0$.

(26) Explain piecewise linear approximations to non-linear functions. Also, write the steps of the computional procedure to solve the approximating problem of a separable programming method.

(27) Use dynamic programming to solve following:

Min $z = x_1^2 + x_2^2 + \dots + x_n^2$

S.t. $x_1 + x_2 + \dots + x_n = b$

and $x_j \geq 0$, $j = 1, 2, \dots, n$

(28) State and prove separating hyperplane theorem.