

PART A

- Q.1. Write relation between Hankel & Laplace Transform.
- Q.2. State validity condition for L.T. of  $\frac{d^n f(x)}{dx^n}$   
Also write value of  $L\left[\frac{d^n f(x)}{dx^n}; p\right]$
- Q.3. State inversion formula for Fourier cosine Transform.
- Q.4. Write Mellin Transform of product of two functions.
- Q.5. Obtain  $H_0\left[\frac{\cos ax}{x}; p\right]$
- Q.6. State and prove change of scale property for Laplace Transform.
- Q.7. Find FCT & FST of  $x^{m-1}$
- Q.8. State and prove convolution theorem for F.T.
- Q.9. Prove that  $M\left[\frac{1}{x} f\left(\frac{1}{x}\right); p\right] = \int_0^\infty x^{p-1} \frac{1}{x} f\left(\frac{1}{x}\right) dx$
- Q.10. Taking  $x J_0(px)$  find infinite H.T. of  $e^{-ax}$ .
- Q.11. Find  $H^{-1}(e^{-ap})$  when  $n=1$

Q.12. Evaluate  $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$   $a > 0, b > 0$

Q.13. Solve I.E  $\int_0^{\infty} f(x) \cos \lambda x dx = e^{-\lambda}$

Q.14. Prove that  $H_\nu(x^{\nu-\frac{1}{2}} e^{-ax}; b) = \frac{2^\nu \Gamma(\nu+\frac{1}{2}) b^{\frac{\nu+1}{2}}}{\sqrt{\pi} (a^2+b^2)^{\frac{\nu+1}{2}}}$

Q.15. Prove Parseval's theorem for Hankel Transform.

Q.16. Prove that

$$M[x^p (1-x)^{c-1} {}_2F_1(a, b; c; 1-x) H(1-x); b] \\ = \frac{\Gamma(c) \Gamma(p+p) \Gamma(p-a-b+c+p)}{\Gamma(p-a+c+p) \Gamma(p-b+c+p)}$$

Q.17. Solve I.E using M.T.  $\int_0^{\infty} f(u) g(xu) du = h(x)$

Q.18. Show that  $M^{-1}\left[\frac{\Gamma(1-p)/2}{\Gamma(2-p)/2} f^*(p); x\right] \\ = \frac{2}{\sqrt{\pi}} \int_0^x \frac{f(t) dt}{\sqrt{x^2-t^2}}$

Q.19. Show that  $M[x^n P_n(x) H(1-x); b] \\ = (p+n)^{-1} {}_2F_1[-n, n+1; n+p+1; \frac{1}{2}]$

Q.20. Find M.T. of  $E_i(x) \in x^m H(x-a)$

PART-B

Q.1. Find LT of  $\sin \sqrt{x}$  and hence obtain LT of  $\frac{\cos \sqrt{x}}{\sqrt{x}}$

Q.2. Explain IVT & BVT.

Q.3. Prove that

$$L\{t^\nu J_\nu(at) ; p\} = \frac{(2a)^\nu \Gamma(\nu + \frac{1}{2})}{\sqrt{\pi} (p^2 + a^2)^{\nu + \frac{1}{2}}}$$

Q.4. Prove that  $\int_0^\infty \frac{J_0(x) - \cos x}{x} dx = \log 2$

Q.5. Find  $L^{-1}\left[\frac{pe^{-2p}}{p^2 + 3p + 2}\right]$

Q.6. Find ILT of  $\frac{1}{\sqrt{p+a}}$

Q.7. Prove that  $\int_0^x J_0(u) \sin(x-u) du = xJ_1(x)$

Q.8. Find  $L^{-1}\left[\frac{11p^3 - 47p^2 + 56p + 4}{(p-2)^3(p+2)}\right]$

Q.9. Find FST & FCT of  $\frac{e^{ax} + e^{-ax}}{e^{\pi x} - e^{-\pi x}}$

Q.10. Prove that  $e^{-x^2/2}$  is self reciprocal function

Q.11. Find FCT of  $\frac{1}{a^2+x^2}$  hence find FST of  $\frac{1}{x(a^2+x^2)}$  and  $\frac{x}{a^2+x^2}$

- Q.12. Show that  $f(x) = x^n$  is of exponential order  $\alpha$  as  $x \rightarrow \infty$
- Q.13. Describe  $E_i(t)$
- Q.14. Write value of  $H_2 \left[ \frac{d^2 f}{dx^2} + \frac{1}{x} \frac{df}{dx} - \frac{v^2}{x^2} f ; p \right]$
- Q.15. Write Parseval's theorem for H.T.
- Q.16. Write sufficient condition for existence of Laplace Transform.
- Q.17. Write value of  $M \left[ x^m f(x^n); p \right]$
- Q.18. Explain self reciprocal function.
- Q.19. If  $H\{f(x)\} = \bar{f}(p)$  then show that  

$$H\{f(ax)\} = \frac{1}{a^2} \bar{f}(p/a)$$
- Q.20. Find H.T. of  $\frac{df}{dx}$  when  $f = \frac{e^{-ax}}{x}$ ,  $n=1$