Department of Mathematics and Statistics

Semester - VI

Paper : Complex Analysis and Mechanics

Part - A

Q.1 Define limit of a complex number

Q.2 Prove that the function $f(z) = \begin{cases} z^2 , & z \neq i \\ 0 , & z = i \end{cases}$

is discontinuous at $z = 0$

Q.3 Prove that continuity is a necessary but not sufficient condition for the existence of a finite derivative

Q.4 Write Polar form of Cauchy - Riemann equations

Q.5 Prove that $\lim_{z \to 0} \frac{\bar{z}}{z}$ does not exist.

Q.6 What is harmonic function

Q.7 What is Cauchy's Integral formula

Q.8 Write the statement of Morera's theorem

Q.9 What is fundamental theorem of integral calculus

Q.10 Write the formula for derivative of an analytic function $f(z)$ at a point 'z_0' inside the closed contour C

Q.11 Write statement of Taylor's Theorem.

Q.12 What is Maximum Modulus Theorem

Q.13 Expand e^z in a Taylor's series about $z = 0$

Q.14 obtain expansion for $\frac{z^2-4}{(z+1)(z+4)}$ which are valid for the region $|z| < 1$

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Q.15 At $z=0$, $f(z) = \dfrac{z-\sin z}{z^3}$ have what kind of singularity

Q.16 Define Removable Singularity

Q.17 Find the kind of singularity of the following function: $f(z) = \dfrac{1}{\sin z - \cos z}$ at $z = \dfrac{\pi}{4}$

Q.18 Find the residue of $\dfrac{z^3}{z^2-1}$ at $z=\infty$

Q.19 Find the residue of $\dfrac{z^2}{(z-1)(z-2)(z-3)}$ at $z=1,2$

Q.20 Explain Meromorphic function with example.

Q.21 Define Radial and Transverse acceleration

Q.22 A particle moves along a circle $r = 2a\cos\theta$ in such a way that its acceleration toward the origin is always zero then prove that

$$\frac{d^2\theta}{dt^2} = -2\cot\theta\left(\frac{d\theta}{dt}\right)^2$$

Q.23 Prove that the acceleration of a point moving in a plane curve with uniform speed is $\rho\dot{\psi}^2$

Q.24 Prove that the angular acceleration in the direction of motion of a particle moving in a plane is

$$\frac{v}{\rho}\frac{dv}{ds} - \frac{v^2}{\rho^2}\frac{d\rho}{ds}$$

Q.25- A particle moves in a curve with constant velocity v. If $s=0$ when $\psi=0$ and at any point its acceleration is $\frac{v^2 c}{s^2+c^2}$, then prove that the curve is a catenary

Q.26 A particle is projected with velocity u along a smooth horizontal plane in a medium whose resistance per unit mass is K(velocity) then show that the velocity v after time t is $v = ue^{-Kt}$.

Q.27 A particle is moving in a straight line under a resistance force which produce retardation to Kv^3. If u is its initial velocity, v be its velocity after time t when it has moved a distance x, then show that $t = \frac{x}{u} + \frac{1}{2}Kx^2$

Q.28 A sphere of given weight W, rests between two smooth planes one vertical and the other inclined at an angle α to the vertical. Find the reactions of the plane on the sphere

Q.29 A rod rests wholly within a smooth hemisphere bowl of radius r, its centre of gravity dividing the rod into two portions 'a' & 'b' show that if θ be the inclination of the rod

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to the horizon in the position of equilibrium then prove that

$$\sin\theta = \frac{b-a}{2\sqrt{r^2-ab}}$$

Q.30 Define angle of friction.

Q.31 Prove that the least force required to pull a body of weight W on a rough horizontal plane is W sin λ, where 'λ' is the angle of friction.

Q.32 What is moment of force. Give geometrical interpretation of the moment.

Q.33 The force 3P, 7P and 5P act along the side AB, BC, CA respectively of an equilateral triangle ABC. Find the magnitude, direction and line of action of their resultant.

Q.34 ABCD is a square having a side of length 3 meters. Forces of 3, 4, 7 & 9 Kg act along the side of the square resp. Find line of action of their resultant.

Q.35 Find the tension in a string or thrust in a rod

Q.36 Two equal uniform rods AB and AC. Each of length 2b are freely joined at A and rest on a vertical circle of radius a. Show that if 2θ be the angle between them then

$$b\sin^3\theta = a\sin\theta$$

Q.37 Find the product of Inertia (P.I) of a uniform rod of mass M and length 2a about two perpendicular axes

Q.38 Obtain Intrinsic equation of the common catenary

Q.39 The end links of a uniform chain of length l can slide on two smooth rods in the same vertical plane which are inclined in opposite directions at equal angle ϕ to the vertical, then prove that.

Sag in the middle $= \frac{1}{2} l \tan\left(\frac{\phi}{2}\right)$

Q.40 A heavy uniform chain of length $2l$ hangs over two small smooth pegs in the same horizontal line at a distance $2a$ apart. Show that if h is the sag in the middle, the length of either part of the chain that hangs vertically is $(h + l - 2\sqrt{hc})$

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Part- B

Q.1 Prove that continuity is a necessary but not sufficient condition for the existence of a finite derivative

Q.2 Show that function $u = \cos x \cosh y$ is a harmonic and find its harmonic conjugate.

Q.3 Prove that $f(z) = \bar{z}$ is not differentiable at any point.

Q.4 Prove that the function

$$f(z) = \sin x \cosh y + i \cos x \sinh y$$

is continuous and analytic everywhere

Q.5 If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and $u - v = e^x (\cos y - \sin y)$, find $f(z)$ in terms of z.

Q.6 Prove that if $f(z)$ is analytic, with a continuous derivative, in a simply connected domain G, and C is a closed contour lying in G, then

$$\int_C f(z)\, dz = 0$$

Q.7 State and prove fundamental theorem of integral calculus for complex functions

Q.8 Find the value $\displaystyle\int_{|z|=1} \frac{\sin^6 z}{(z - \pi/6)^3}\, dz$

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Q.9 Expand e^z and $\sin z$ in a Taylor's series about $z=0$ and determine the region of convergence in each case.

Q.10 Find different expansion of $\frac{1}{(z-1)(z-3)}$ in power of 'z' which are valid for regions.

(i) $|z|<1$ (ii) $1<|z|<3$

Q.11 By considering the Laurent series for

$$f(z) = \frac{1}{(1-z)(z-2)}$$

then prove that if 'C' be any closed contour within the annulus

$$1<|z|<2, \quad \int_C f(z)\,dz = 2\pi i$$

Q.12 If $f(z) = \frac{1}{z^4+2z^2+1}$, then show that $f(z)$ has two double poles

Q.13 State and prove Riemann's theorem

Q.14 Find out the zeros and discuss the nature of singularities of $f(z) = \frac{z-2}{z^3} \sin \frac{1}{z-1}$

Q.15(a) Prove that the residue of functions $\frac{z}{(z-a)(z-b)}$ and $\frac{z^3-z^2+1}{z^3}$ at infinity are -1 and 1 respectively

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Q.15 (b) Find the residue of the function.

$$f(z) = \frac{1}{\sinh z}$$ at the pole $z = i n\pi$.

Q.16 Prove that a function $f(z)$ is a polynomial of degree 'n' if and only if $f(z)$ has no singularities in the finite part of the plane and has a pole of order n at infinity

Q.17 If the position of a moving particle in a plane at time t is given by $x = 8t-1$, $y = 8t^2$ meter, find the velocity and acceleration of particle at $t = 3$ second

Q.18 Velocity of a moving particle in a plane parallel to the x-axis and y-axis are $u + ey$ and $u + ex$ respectively. Show that path of the particle is a conic section

Q.19 A particle describes equiangular spiral $r = ae^{m\theta}$ with constant speed. Find the radial and transverse components of its velocity and acceleration

Q.20 A regular hexagon ABCDEF consists of six equal ~~rods~~ rods which are each of weight W and are freely jointed together. The hexagon rests in a vertical plane and AB is in contact with a horizontal table. If C and F are connected by a

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light string. Prove that its tension is $W\sqrt{3}$

Q.21 Forces P, Q, R act along the sides BC, AC and BA resp. of an equilateral triangle ABC. If their resultant is a force parallel of BC through the centroid of the triangle. Prove that $Q = R = \frac{1}{2}P$

Q.22 Prove Varignon's theorem for moments.

Q.23 A heavy carriage wheel of weight W and radius r is to be dragged over an obstacle of height h by a horizontal force F applied to the centre of the wheel. Show that F must be slightly greater than $\frac{W\sqrt{2hr-h^2}}{r-h}$

Q.24 A particle of mass m is falling under gravity through a medium whose resistance is μ times the velocity. If the particle is released from rest. Show that the distance fallen through in time t is:
$$\frac{gm^2}{\mu^2}\left[e^{-\mu t/m} - 1 + \frac{\mu t}{m}\right]$$

Q.25 A particle of mass 'm' is projected with velocity u along a smooth horizontal plane in a medium whose resistance is mKv^3. Show that the distance it has described in time t is
$$\frac{1}{Ku}\left\{\left(1+2Ku^2t\right)^{1/2}-1\right\}.$$

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Q.26 How high can a particle rest inside a hollow sphere of radius 'a'. If the coefficient of friction be $\frac{1}{\sqrt{3}}$

Q.27 A body of weight 100 Kg is situated on a rough plane, where the angle of friction is 30. Find the minimum force, which may bring it in motion.

Q.28 The least force which will move a weight up an inclined plane is P. Show that the least force parallel to the plane which will move the weight up the plane is $P\sqrt{1+\mu^2}$, where μ is the angle of friction.

Q.29 The moments of a system of coplanar forces (not in equilibrium) about three collinear points A, B, C in the plane are G_1, G_2, G_3. Prove that $G_1 \cdot BC + G_2 \cdot CA + G_3 \cdot AB = 0$

Q.30 Show that the length of an endless chain which will hang over a circular pulley of radius 'a' so as to be in contact with two third of the circumference of the pulley is

$$a\left[\frac{4\pi}{3} + \frac{3}{\log(2+\sqrt{3})}\right]$$

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Part - C

Q1 Prove that necessary condition that a function $f(z) = u(x,y) + i v(x,y)$ be analytic in a domain D is that u and v satisfy the Cauchy- Riemann equation i.e. $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$, $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

Q2 If $f(z) = u + iv$ is an analytic function, where both $u(x,y)$ and $v(x,y)$ are conjugate functions, given one of these say $u(x,y)$ to find the other $v(x,y)$

Q3 If $f(z) = u + iv$ is an analytic function of $z = x + iy$ and

$$u - v = \frac{\cos x + \sin x - e^{-y}}{2\cos x - e^{y} - e^{-y}}$$

Q.4 Show that the function f defined by.

$$f(z) = e^{-z^{-4}}, \quad z \neq 0 \quad \text{and} \quad f(0) = 0$$

is not analytic at $z=0$, although Cauchy- Riemann equation are satisfied.

Q.5 Prove that derivative of an analytic function is itself an analytic function

Q6 Evaluate the integral $\displaystyle\int_{|z|=3} \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$

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Q.7 State and prove Laurent's theorem

Q.8 Let $f(z)$ be ~~andy~~ analytic within and on a simple closed contour C. Let $|f(z)| \le M$ on C. Then $|f(z)| < M$ for every point z within C and if $|f(z)| = M$ for some z within C then prove that $|f(z)| = M$ for all z for all z within and on C.

Q.9 Discuss the nature of singularities of the function

$$f(z) = \frac{e^{c/(z-a)}}{e^{z/a} - 1}$$

Q.10 Prove Cauchy's Residue theorem

Q.11 Prove that an entire function $f(z)$ whose singularity at infinity is at the most, a pole is necessarily a polynomial

Q.12 The radial and transverse velocities of a particle are λr and $\mu\theta$. Find its path and show that its radial and transverse components of acceleration are respectively

$$\lambda^2 r - \frac{\mu^2\theta^2}{r} \quad \text{and} \quad \mu\theta\left(\lambda + \frac{\mu}{r}\right)$$

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Q.13 If the angular velocity of a point moving in a plane curve be constant about a fixed point. Show that its transverse acceleration varies its radical velocity

Q.14 A particle is projected in a medium whose resistance is proportional to the cube of the velocity and no other force act on the particle. While the velocity diminishes from v_1 to v_2 and particle traverse a distance d in time t, show that

$$\frac{d}{t} = \frac{2v_1 v_2}{v_1 + v_2}$$

Q.15 A particle of mass 'm' is projected vertically upward with the velocity u under gravity in a resisting medium. Whose resistance varies as the velocity. Discuss the motion

Q.16 The moments of a system of coplanar forces (not in equilibrium) about three collinear point A, B, C in their plane are G_1, G_2, G_3 resp. Prove that

$$G_1 \cdot BC + G_2 \cdot CA + G_3 \cdot AB = 0$$

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Q.17 Prove that a system of coplaner forces acting at different points of a rigid body, which is not in equilibrium, can be reduced either to a single forces or to a single couple.

Q.18 A uniform heavy rod rest in limiting equilibrium within a rough hollow sphere. If rod subtend an angle 2α at the centre of the sphere and if λ be the angle of friction. Show that the angle of inclination of the rod to the horizon is

$$\tan^{-1}\left\{\frac{\sin 2\lambda}{\cos 2\alpha + \cos 2\lambda}\right\}$$

Q.19 A uniform chain of length l, is to be suspended from two point A and B in the same horizontal line so that either terminal tension is n times that at the lowest point, show that

$$\text{span} = AB = \frac{l}{\sqrt{n^2-1}} \log_e\left[n + \sqrt{(n^2-1)}\right]$$

Q.20 A uniform ladder of length l and weight w is resting with one end on a rough horizontal ground and the other end against a smooth vertical wall. If the coefficient of friction between the ladder and the ground is 0.25, show that distance 'x' a man of weight W can climb up the ladder without ladder slipping is given by $x = \frac{1}{4}\left(1+\frac{w}{W}\right) l \tan\theta = \frac{Wl}{2w}$