B.Sc Part-III

Mathematics-III

D Mechanics

Q.1. A body moving in a straight line OAB with SHM has zero velocity when at points A & B whose distances from O are a and b respectively and has a velocity v when half way between them. Show that complete time period is $\frac{\pi(b-a)}{v}$

Q.2. A particle of mass m is attached to a light wire which is streched between two fixed point with Tension T. If a & b are the distances of the particle from two ends, prove that period of a small transverse osscillation is :- $2\pi\sqrt{\frac{mab}{T(a+b)}}$

Q.3. A particle moves along the curve $y = a \log \sec\left(\frac{x}{a}\right)$ in such a way that the tangent to the curve rotates uniformly. Prove that the resultant acceleration of the particle varies as the square of radius of curvature.

Q.4. The radial and transversal velocities of a particle are λr & $\mu\theta$. Find its path and show that the radial and transversal components of accelerations are $\lambda^2 r - \frac{\mu^2\theta^2}{r}$ and $\mu\theta\left(\lambda + \frac{\mu}{r}\right)$.

Q.5 A particle of mass m is projected vertically upwards under gravity, the resistance of air being mk times the velocity. Show that the greatest height attained by the particle is $\frac{V^2}{g}[\lambda - \log(1+\lambda)]$ where V is terminal velocity of particle and λV is initial velocity. Also show the corresponding time is $\frac{V}{g}\log(1+\lambda)$.

Q.6. A cannon ball has a range R on the horizontal plane. If h and h' be the greatest height in the two paths for which this is possible, prove that

$$R = 4\sqrt{hh'}$$

Q.7. A heavy particle of weight W, attached to a fixed point by a light inextensible string, describes a circle in a vertical plane. The tension of the string has values mW and nW respectively, when the particle is at heighest and lowest point of its path. Show that $n = m + 6$

Q.8 A bullet looses $\frac{1}{15}^{th}$ of its velocity in passing through a plank. Find with the help of principle of work and energy, how many such uniform planks be required to bring bullet to rest

Q.15. A semi circular disc rests in a vertical plane with its curved edge on a rough horizontal and an equally rough vertical plane, the coefficient of friction being μ. Show that the greatest angle which the bounding diameter can make with the horizontal plane is $\sin^{-1}\left[\frac{3\pi}{4}\frac{\mu^2+\mu}{1+\mu^2}\right]$

Q.16 Show that length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with two thirds of the circumference of the pulley is

$$a\left[\frac{4\pi}{3}+\frac{3}{\log 2+\sqrt{3}}\right]$$

Q.17. A string of length a, forms the shorter diagonal of a rhombus formed of four uniform rods, each of length b and weight W which are hinged together. If one of the rods be supported in a horizontal position, prove that the tension of string is

$$\frac{2W(2b^2-a^2)}{b\sqrt{4b^2-a^2}}$$

Q.18. Prove that the least force required to pull a body of weight W on a rough horizontal plane is $W\sin\lambda$, where λ is angle of friction.

Q.9 A particle describes $r^n \cos n\theta = a^n$ under a central force P. Show that $P \propto r^{2n-3}$

Q.10 A planet describes an ellipse about the sun as focus. Prove that its velocity at the end of the minor axis is geometric mean between its velocities at the ends of any diameter

Q.11 Find M.I of hollow sphere of radius a and mass M about a diameter

Q.12. Show that M.I of semi circular lamina about a tangent parallel to the bounding diameter is $Ma^2\left(\frac{5}{4} - \frac{8}{3\pi}\right)$ where a is the radius and M is mass of lamina.

Q.13. Forces 1, 3, $5\sqrt{2}$ kg wt act along the sides AC, AB, BC resp. of an isosceles triangle ABC right angled at A. Find the magnitude, direction and line of action of resultant

Q.14. A uniform beam of length 2a, rests in equilibrium position against smooth vertical wall and over a smooth peg at a distance b from wall. If θ be inclination of the beam to the vertical, show that

$$\sin^3\theta = b/a$$

Q.19. A square of side $2a$ is placed with its plane vertical between two smooth pegs which are in the same horizontal line and at a distance c apart, show that it will be in equillibrium when the inclination of one of its edges to the horizon is either 45° or $\frac{1}{2}\sin^{-1}\left\{\frac{a^2-c^2}{c^2}\right\}$

Q.20 A heavy uniform chain of length $2l$ hangs over two small smooth pegs in the same horizontal line at distance $2a$ apart. Show that if h is the sag in the middle, the length of either part of the chain that hangs vertically is $h+l-2\sqrt{hl}$