Department of Mathematics

Semester - III

Real Analysis - I and Differential Equations - I

Part - A

Q.1. Show that every open interval $]a,b[$ is a n.b.d of each of its points.

Q.2. Find limit points of the set $S = \{\pm 1, \pm \frac{3}{2}, \pm \frac{4}{3} \cdots\}$

Q.3. If $I_n =]x - \frac{1}{n}, x + \frac{1}{n}[$ $n \in \mathbb{N}$, then show that $\bigcap_{n=1}^{\infty} I_n$ is not an open set.

Q.4. Find the limit points of the sequence $\langle (-1)^n (1 + \frac{1}{n}) \rangle$

Q.5. Show that the sequence $\langle \frac{1}{n} \rangle$ is a cauchy sequence.

Q.6. Show that the function $f(x) = \begin{cases} x, & \text{if } x \text{ is rational} \\ 1-x, & \text{if } x \text{ is irrational} \end{cases}$ is continuous at $x = \frac{1}{2}$.

Q.7. Show that $f(x) = x^2$ is uniformly continous on $[-2, 2]$

Q.8. Define connected set.

Q.9. Prove that Every convergent sequence is bounded.

Q.10. Examine whether the sequence $\{\frac{1}{n^3}\}$ is convergent or not?

Q.11. State Cauchy's first theorem on limit.

Q.12. Define infimum and supremum of function.

Q.13. What is Heine's definition of continuity.

Q.14. State mostest theorem.

Q.15. Define uniform continuity.

Q16. State Bolzano - weristrass theorem.

Q.17. State general principal of convergence of a sequence

Q.18 Define limit point of sequence.

Q.19. State Heine Borel theorem.

Q.20. Define compact set

Q.21. Define order and degree of a differential equation.

Q.22 Find the order and degree of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^{3/2} = \left(\frac{dy}{dx}\right)^3$$

Q.23. Solve the differential eq

$$y\,dx + x\,dy + \frac{x\,dy - y\,dx}{xy} = 0$$

Q.24. Write P.I of $(D^2+4D-12)y = e^{3x}$

Q.25. Find C.F of $(x^2D^2 + xD - 1)y = x^2$

Q.26. Find one part of CF of D.E

$$x^2\frac{d^2y}{dx^2} - (x^2-2x)\frac{dy}{dx} + (x+2)y = x^3e^x$$

Q.27. Find general solution of the following equation

$$y = px + \frac{a}{p}$$

Q.28. Define general solution, singular solution and extraneous loci of ode.

Q.29. Solve $(D^4 - a^4)y = 0$

Q.30 Define homogeneous linear differential equation

Q.31. Define exact differential equation and write necessary and sufficient condition for $Mdx+Ndy=0$ to be exact.

Q.31. Write the formula for general solution of $Mdx + Ndy = 0$.

Q.31. Solve $\sin px \cos y = \cos px \sin y + p$

Q.32. How do you find PI for $xV(x)$ type function

Q.35.

Define differe

Write clairaut's equation and its solution.

Define with example first order first degree linear differential equation.

Q.35. Define ode and pde.

Q.36. Form differential eqn for S.H.M $x = a\cos(nt+b)$.

Q.37. Find degree of eqn $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} = \int x^2 dx$

Q.38. Solve $(1-x)dy - (3+y)dx = 0$

Q.39. Write standard form of linear differential eqn of order one. and find its integrating factor.

Q.40. Find IF of $(1+y^2)dx = (\tan^{-1}y - x)dy$.

Part – B

Q.1. Show that $\langle \frac{1}{n^3} \rangle$ is a cauchy sequence.

Q.2. Prove that the necessary and sufficient condition for the $\langle x_n \rangle$ to be a convergent sequence is that its a cauchy sequence

Q.3. Prove that the sequence $\langle x_n \rangle$ where $x_1 = \frac{1}{2}$, and $x_{n+1} = \frac{2x_n+1}{3}$, $n \in N$ is convergent. Also find its limit.

Q.4. Prove that derived set of any set is a closed set.

Q.5. Prove that the set R of real numbers is not a compact set.

Q.6. Prove that the necessary and sufficient condition for a monotonically increasing sequence $\langle x_n \rangle$ to be convergent is that it is bounded and in that case $\lim x_n$

Q.7. Prove that intersection of finite number open set is open.

Q.8. Prove that a set is closed iff its Complement is open.

Q.9. Prove that every bounded and infinite set has atleast one limit point.

Q.10. Prove that necessary and sufficient condition for a sequence $<x_n>$ to be a convergent sequence is that $<x_n>$ is bounded and has a unique limit point.

Q.11. State and prove Bolzano-Weierstrass theorem.

Q.12. Prove that the union of an arbitrary collection of open sett is again open.

Q.13. State and prove Heine-Borel theorem.

Q.14. Prove that product set $N \times N$ is countable set.

Q.15 Prove that any subset of real numbers is connected set iff it is an interval

Q.16. Using Cauchy's general principal of convergence for sequences, Prove that $x_n = 1 + \frac{1}{2} + \frac{1}{3} + --- \frac{1}{n}$ is convergent.

Q.17. State and prove intermediate value theorem.

Show that $f(x) = \frac{1}{x^2}$ is uniformly continuous on $[a, \infty)$, $(a>0)$ but not uniformly continuous on $(0, \infty)$.

Q.19. Prove that if a function is continuous on $[a, b]$, then it is bounded in the interval.

Q.20. If $f(x) = \frac{x^n}{1+x^n e^x}$ the find the type of discontinuity at $x=1$

Q.21. Find the differential equation of all circles which passes through origin and and having centre at x-axis.

Q.21. Solve $(x+y)^2 \frac{dy}{dx} = a^2$

Q.22. Solve $(1+e^{x/y})\,dx + e^{x/y}\left\{1 - \frac{x}{y}\right\} dy = 0$

Q.23. Solve $\frac{dy}{dx} = \frac{x+y+1}{2x+2y+3}$

Q.24. Solve $\frac{dy}{dx} + \frac{y}{(1-x)\sqrt{x}} = 1-\sqrt{x}$

Q.25. Solve $\cos x\, dy = y(\sin x - y)\, dx$

Q.26. Solve $(x^2y^2 + 5xy + 2)\, y\,dx + (x^2y^2 + 4xy + 2)\, x\,dy = 0$

Q.27. Solve $y - 2px = f(xp^2)$

Q.28. Solve $x\left(\frac{dy}{dx}\right)^2 + (y-x)\frac{dy}{dx} - y = 0$

Q.29. Solve $y = 2px + y^2p^3$

Q.30. Find the general solution, singular solution and extraneous loci:

Q.31. Solve $(D^2+4)y = \tan 2x$

Q.32. Solve $(D^2+9)y = \cos 2x + \sin 2x$

Q.33. Solve $(D^3 - D^2 - 6D)y = (x^2-1)$

Q.34. Solve $x^4 \frac{d^3y}{dx^3} + 2x^3 \frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = 1$

Q.35. Solve $\frac{d^3y}{dx^3} + \cos x \frac{d^2y}{dx^2} - 2\sin x \frac{dy}{dx} - y\cos x = \sin 2$

Q.36. Solve $x^4 \frac{d^2y}{dx^2} + x^2(x-1) \frac{dy}{dx} + xy = x^3 - 4$

Q.37. Solve $\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} + xy = x$

Q.38. Solve $\sin^2 x \frac{d^2y}{dx^2} = 2y$

Q.39. Solve $\frac{d^2y}{dx^2} + 2\tan x \frac{dy}{dx} + y(1+2\tan^2 x) = \sec x \tan x$

Q.40. Solve $\frac{d^2y}{dx^2} + \frac{2}{x}\frac{dy}{dx} + \frac{a^2}{x^4} y = 0$

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