Department of Mathematics

Semester - IV

Real Analysis-II and Numerical Analysis

Part - A

Q.1. Find the value of $\Delta \tan^{-1} x$

Q.2. Write the fundamental theorem of difference calculus.

Q.3. Prove that $E = 1 + \nabla$, and $\nabla = 1 - E^{-1}$

Q.4. Prove that $\Delta \log f(x) = \log \left[1 + \frac{\Delta f(x)}{f(x)}\right]$

Q.5. Find value of $\Delta (e^{2x} \log 3x)$

Q.6. Prove that $\Delta - \nabla = \Delta \nabla$

Q.7. Write the newton gregory forward difference formula.

Q.8. Write the newton gregory backward difference formulae.

Q.9. Write Lagrange's interpolation formula.

Q.10. Write Newton's divided difference formula.

Q.11. If $f(x) = \frac{1}{x}$ show that $\underset{b}{\Delta} f(a) = -\frac{1}{ab}$

Q.12. Write general quadrature formula for numerical integration.

Q.13. Write gauss quadrature formula for numerical integration.

Q.14. What is signifance of numerical analysis.

Q.15. Write formula for Newton Raphson method.

Q.16 Why is Euler's method used?

Q.17. Write regula falsi method.

Q.18 Prove that given function is continuous at $x=0$, but not differentiable.

$$f(x)=\begin{cases} x\sin\frac{1}{x} & x\neq 0 \\ 0 & x=0 \end{cases}$$

Q.19. Define derivative of inverse function

Q.20 State Darboux theorem.

Q.21. State Rolle's theorem.

Q.22. Find directional derivative of $f(x,y)=x+y^2$ at the point $(4,0)$ in the direction $\hat{x}=\frac{1}{2}\hat{i}+\frac{\sqrt{3}}{2}\hat{j}$

Q.23. If $u = xy\,f(y/x)$ then prove that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=2u$$

Q.24. If $x^3+y^3=3axy$ then find $\frac{dy}{dx}$.

Q.25. Define partition and norm of a partition of an interval.

Q.26. If $f(x,y)=\frac{xy}{x^2+y^2}$ then show that $\lim_{(x,y)\to(0,0)} f(x,y)$ doesn't exist.

Q.27. Define refinement of a partition.

Q.28. Define Darboux Sum.

Q.29. State Darboux theorem.

Q.30. Define upper and lower Riemann integrals.

Q.31. Write necessary and sufficient condition for R-integrability.

Prove that $f(x) = [x]$ is R- integrable in $[0,-3]$

Q.33 Write fundamental theorem of integral calculus.

Q.34. Write first mean value theorem of integral calculus.

Q.35. Write second mean value theorem of integral calculus.

Q.36. Define functions of bounded variation.

Q.37. If $f: [a,b] \to R$ is a function of bounded variation on $[a,b]$ then show that $|f|$ is a function of bounded variation on $[a,b]$.

Q.38. Let f is defined on the closed interval $[a,b]$ in following manner :-

$$f(x) = \begin{cases} 1 & x \in Q \\ -1 & x \in R-Q \end{cases}$$

then for any partition P of the interval $[0,1]$, find $L(P,f)$ and $U(P,f)$.

Q.39. Show that $\lim_{\substack{x \to 0 \\ y \to 0}} \tan^{-1}\left(\frac{y}{x}\right)$ doesn't exist.

Q.40. Prove that integral function is continuous

Part - B

Q.1. Prove that the function $f(x,y) = xy$ is continuous at each point of the xy-plane.

Q.2. State and prove Darboux theorem.

Q.3. Test the continuity and differentiability $f(x) = |x-2| + 2|x-3|$ at $[1,4]$.

Q.4. State and prove Rolle's theorem.

Q.5. Explain geometrical interpretation of Rolle's theorem.

Q.6. Show that $f(x,y) = \begin{cases} \dfrac{x^3 - y^3}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$

is continuous but not differentiable at $(0,0)$

Q.7. If $x^3 + y^3 - 3ax^2 = 0$ then prove that

$$\frac{d^2y}{dx^2} + \frac{2a^2x^2}{y^5} = 0$$

Q.8. Show that between any two roots of $e^x \cos x = 1$ there exist at least one root of $e^x \sin x - 1 = 0$

Q.9. Show that $\dfrac{x}{1+x} < \log(1+x) < x \quad (x>0)$

Q.10. Explain derivatives and give an example of a function which is continuous but not differentiable.

Q.11. Prove that $f(x) = \begin{cases} 0, & \text{If } x \text{ is rational} \\ 1, & \text{If } x \text{ is irrational} \end{cases}$

~~them~~ is not R-integrable on $[0,1]$.

Q.12. Discuss necessary and sufficient condition for R-integrability of a function.

Q.13. If $f(x) = x$, $x \in [0,1]$, then show that f is R-integrable on $[0,1]$ and that

$$\int_0^1 x\,dx = \frac{1}{2}$$

Prove that $\frac{1}{\pi} \leq \int_0^1 \frac{\sin \pi x}{1+x^2} dx \leq \frac{2}{\pi}$

15. Show that $f(x) = \begin{cases} x \sin \frac{\pi}{x} & \text{If } x \in (0,1] \\ 0 & \text{If } x = 0 \end{cases}$

is not a bounded variation.

Q16 Find the total variation of the function $\sin x$ in $[0, 2\pi]$

Q.17. If $f : [a, b] \rightarrow R$ be monotonic, then prove that f is bounded variation over $[a, b]$ and $\overset{b}{\underset{a}{V}}(f) = |f(b) - f(a)|$.

Q.18. Prove the fundamental theorem of integral calculus.

Q.19. State and prove first mean value theorem of integral calculus.

Q.20. State and prove second mean value theorem of integral calculus.

Q.21. Establish the relation between Δ, ∇, E and D.

Q.22. Show that $\sum_{k=0}^{n-1} \Delta^2 f_k = \Delta f_n - \Delta f_0$

Q.23. Find the cubic polynomial which takes the following data :-

x	0	1	2	3
$f(x)$	1	0	1	10

Q.24. The population of a town is decennial census was given as below:-

years :	1891	1901	1911	1921	1931
Population :	46	66	81	93	101

Q.22. If $f(x) = \frac{1}{x^2}$, then $f(a,b)$, $f(a,b,c)$ and $f(a,b,c,d)$

Q.23. Find the value of $y(10)$ with the help of following data :-

x :	5	6	9	11
y :	12	13	14	16

using divided difference formula.

Q.24. Find the polynomial satisfying $(-4, 1245)$, $(-1, 33)$, $(0, 5)$, $(2, 9)$ $(5, 1335)$.

Q.25. Find $f'(1)$ for $f(x) = \frac{1}{1+x^2}$ using following data :-

x	1	1.1	1.2	1.3	1.4
$f(x)$	.25	.2268	.2066	.1890	.1736

Q.26. Compute the value of following integral by Trapezoidal rule $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)\, dx$

Q.27. Calculate the value of integral $\int_{4}^{5.2} \log_e x \, dx$ using simpson's $\frac{1}{3}$rd rule.

Q.28. Evaluate $\int_{0}^{1} \frac{dx}{1+x^2}$ using simpson's $\frac{3}{8}$th rule.

Q.29. Using Gauss three point quadrature formula compute the integral $\int_{-12}^{12} \frac{dx}{24+x}$

Q.30. Derive general quadrature formula.

Q.31. Using bisection method, find real root of the equation $x^4 + 2x^3 - x - 1 = 0$

Q.32. Using method of false position method find root of equation $x^x = 100$.

Q.33. Find by method of iteration a real root the following equation

$$2x - \log_{10} x = 7$$

Q.34. Find the real root of equation $x^3 - 3x - 5 = 0$ using Newton Raphson method.

Q.35. Using Euler's method, find the solution of IVP $\frac{dy}{dx} = xy$ $y(0) = 1$ at $x = 0.4$ with $h = 0.1$

Q.36. Using modified euler's method find solution of $\frac{dy}{dx} = 2 + \sqrt{xy}$ $y(1) = 1$, at $x = 1.6$ with $h = 0.2$

Q.37. Using Newton Raphson method find value of $\sqrt{13}$.

Q.38. Find missing value in following table:-

x	0	1	2	3	4
y	1	3	9	?	81

Q.39. Prove that

$$\frac{\Delta^2}{E} \sin(x+h) + \frac{\Delta^2 \sin(x+h)}{E \sin(x+h)} = \frac{(2\cosh - 1)}{[\sin(x+h)+1]}$$

Q.40. Evaluate $\Delta^3 \left[\frac{1}{(3x+1)(3x+4)(3x+7)} \right]$.
