B.Sc. Part - II

Mathematics - I
Real Analysis

Q.1 Express the set of real numbers R as a complete ordered field.

Q.2. Define compact set and prove that set R of real numbers is not compact.

Q.3 Define semi metric and give an example of a semi metric which is not a metric.

Q.4. Prove that every open sphere in a metric space X is an open set.

Q.5 Define limit and limit point of a sequence and give an example of sequence doesn't have a limit but limit points.

Q.6 If $\{x_n\}$ is a real sequence which converges to l then prove that every subsequence $\{x_{n_k}\}$ of the sequence $\{x_n\}$ also converges to l

Q.7. Show that real sequence defined by $x_n = 1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \cdots \frac{(-1)^{n-1}}{n^2}$ is convergent.

Q.8 Prove that every bounded function need not to be R- integrable

Q.9. If f' is exist for a f and is bounded on $[a,b]$ then prove that the function f is of Bounded variation on $[a,b]$

Q.10 If f is real valued function defined and and continuous on $[a,b]$ then prove that f is R- integrable on $[a,b]$

Q.11. If f is real valued defined and R-Integrable on $[a,b]$. Then there exist a real number μ lying between bounds of f such that

$$\int_a^b f(x)\,dx = \mu(b-a)$$

Q.12. Let f be a continuous function defined on $[a,b]$ such that $f(x) \in [a,b]$ for each $x \in [a,b]$ then prove that f has a fixed point, i.e. $\exists$ a point $x_0 \in [a,b]$ s.t. $f(x_0) = x_0$

Q.13. $f_n(x) = \dfrac{nx}{1+n^2x^2}$ $\forall\, n \in N,\ x \in [0,1]$

Test uniform convergence of sequence $\{f_n\}$

Q.14. Show that series $\sum_{n=1}^{\infty} u_n(x)$ whose sum of first n terms is given by $f_n(x) = nx(1-x)^n$ $n \in N$, $x \in [0,1]$ can be integrated term by term whereas it is not uniformly convergent in $[0,1]$

Q.15. Show that function f defined by

$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}} & \text{if } x^2+y^2 \neq 0 \\ 0 & \text{if } x^2+y^2 = 0 \end{cases}$$

posseses partial derivatives but not differentiable at origin

Q.16. Show that between any two roots of eqn $e^x \cos x - 1 = 0$ there exist at least one root of eqn $e^x \sin x - 1 = 0$

Q.17. If $f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^4} & (x,y) \neq (0,0) \\ 0 & (x,y) = 0 \end{cases}$

the find directional derivative of f along $u = (\sqrt{2}, \sqrt{2})$ at $(0,0)$

Q.18. State and prove rolle's theorem.

Q.19. A subset A of a metric space A is closed iff it contains all of its limit points

Q.20. Prove that function $f(x) = \cancel{}\, x \cos \frac{\pi x}{2}$

$f(0) = 0$ is bounded but not a bounded variation.