B.Sc. Part - II

PAPER III

Numerical Analysis and vector Calculus

Q1 State and prove fundamental Theorem of difference calculus.

Q2. Prove that $(n+1)\Delta^n 0^n = 2\left(\Delta^{n-1}0^n + \Delta^n 0^n\right]$

Q3. Show that $\sum_{K=0}^{n-1} \Delta^2 f_k = \Delta f_n - \Delta f_0$

Q4. From following Data estimate the numbers of person earning wages between Rs 60 & 70

Wages:	below 40	40-60	60-80	80-100	100-120
no of persons: (in thousand)	250	120	100	70	50

Q5. The population of a town in decennial census was given below:-

year	1891	1901	1911	1921	1931
population (in thousands)	46	66	81	93	101

Estimate population for the year 1925.

Q6. by means of Lagrange's formulae prove that

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left(\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right)$$

Q7. Find polynomial satisfied by

(-4, 1245) (-1, 33) (0, 5) (2, 9) (5, 1335)

Q.8. Given the following data, find $f(x)$ as a polyn of in powers of $(x-5)$

x	0	2	3	4	7	9
$f(x)$	4	26	58	112	466	922

Q.9. With usual notation prove that

(i) $\mu^2 = \frac{1}{4}(\delta^2+4)$

(ii) $\mu\delta = \frac{1}{2}(\Delta+\nabla)$

Q.10. Using stirling formulae find y_{11} from data given below:-

$y_0 = 3010$, $y_5 = 2710$, $y_{10} = 2285$, $y_{15} = 1860$, $y_{20} = 1560$, $y_{25} = 1510$, $y_{30} = 1835$

Q.11. Using gauss interpolation formulae find $f(41)$ with the data

x	30	35	40	45	50
$f(x)$	3678.2	2995.1	2400.1	1876.2	1416.3

Q12. Find $f'(5)$ from following data

x	0	2	3	4	7	9
$f(x)$	4	26	58	112	466	922

Q.13. Find fisst two derivatives of $f(x)$ at $x=1$ from given data:-

x	-2	-1	0	1	2	3	4
$f(x)$	104	17	0	-1	8	69	272

Q.14. Calculate by simpson's $\frac{1}{3}$ rule an approx. Value of $\int_{-3}^{3} x^4 dx$ by taking seven equidistant ordinates. Compare it with exact value and the value obtained by using traperoidal rule.

Q.15 using Gauss three point quadrature formulae Compute $\int_0^1 \sqrt{1+2x}\, dx$

Q.16. Find to four places of decimal the smallest root of eqⁿ $e^{-x} - \sin x = 0$ using newton Raphson method.

Q.17. Find root of eqⁿ $x^x = 100$ by regula falsi method.

Q.18. Solve following system of linear eqⁿ using Gauss siedel method:-

$$27x_1 + 6x_2 - x_3 = 85$$
$$6x_1 + 15x_2 + 2x_3 = 72$$
$$x_1 + x_2 + 54x_3 = 110$$

Q.19. Using Euler's modified method find solⁿ of $\frac{dy}{dx} = 2 + \sqrt{xy}$, $y(1) = 1$ at $x = 1.6$ with $h = .2$

Q.20. Using picard method solve $\frac{dy}{dx} = 1+xy$ $y(1)=2$ upto third approx.

Q.21. If sum of two roots of eq^n $x^4+4x^3-2x^2-12x+9=0$ is zero then solve the eq^n.

Q.22. If α, β, γ be roots of eq^n $x^3-px^2+qx-r=0$ find values of symmetric functions :-
(i) $\Sigma \alpha^2\beta$ (ii) $\Sigma \alpha^3$ (iii) $\Sigma \alpha^2\beta^2$

Q.23. If eq^n $x^5-10a^3x^2+b^4x+c^5=0$ has three equal roots then show that
$ab^4-9a^5+c^5=0$

Q.24. Solve eq^n $x^3+6x^2+9x+4=0$ using cardon's method.

Q.25. Solve eq^n $x^4-2x^2+8x-3=0$ using ferrari method.

Q.26. Verify green's theorem in the plane for $\int_C x^2-xy^3\,dx+(y^2-2xy)\,dy$ where C is the square with vertices (0,0), (2,0), (2,2), (0,2)

Q.27. Evaluate $\int_C e^x\,dx+2y\,dy-dz$ by stoke's theorem where C is curve $x^2+y^2=4$, $z=2$

Q.28. Evaluate $\int_S (x\hat{i}+y\hat{j}+z^2\hat{k})\cdot \hat{n}\,ds$ where S is surface bounded by cone $x^2+y^2=z^2$, plane $z=1$

Q.29. Prove that $\nabla \times (\nabla \times \vec{a}) = \nabla(\nabla \cdot \vec{a}) - \nabla^2 \vec{a}$

Q.30. Prove that

$\text{Curl}(\vec{a} \times \vec{b}) = (\vec{b} \cdot \nabla)\vec{a} - \vec{b}\,\text{div}\,\vec{a} - (\vec{a} \cdot \nabla)\vec{b} + \vec{a}\,\text{div}\,\vec{b}$

Q.31. Prove that

$\text{div}(\vec{a} \times \vec{b}) = \vec{b} \cdot \text{curl}\,\vec{a} - \vec{a} \cdot \text{curl}\,\vec{b}$

Q.32. Using green's theorem find area of ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Q.33. Solve system of linear eqn using Gauss elimination method :-

$x - y + 2z = 3$

$x + 2y + 3z = 5$

$3x - 4y - 5z = -13$

Q.34. Find the value of $(10)^{1/3}$ by newton raphson method

Q.35. Using simpson '$\frac{3}{8}$' rule evaluate $\int_0^1 \frac{x}{1+x}\,dx$

Q.36 Solve $\int_0^1 \frac{\arctan x}{x^{3/2}}\,dx$

Q.37. Prove that $u_1 x + u_2 x^2 + u_3 x^3 + \text{- - - -}$

$= \frac{x}{1-x} u_1 + \frac{x^2}{(1-x)^2} \Delta u_1 + \frac{x^3}{(1-x)^3} \Delta^2 u_1 + \text{- - -}$