

B.Sc. (part II) - Mathematics

paper II - Differential equations and partial differential equations

1. a) (i) find the order and the degree of the following differential equation:

$$\left[ y + x \left( \frac{dy}{dx} \right)^2 \right]^{4/3} = x \frac{d^2y}{dx^2}$$

ii) solve:

$$(e^y + 1) \cos x dx + e^y \sin x dy = 0$$

2. a) solve:  $(1 + y^2) dx = (\tan^{-1} y - x) dy$

b) solve:  $x dx + y dy = a^2 \left( \frac{x dy - y dx}{x^2 + y^2} \right)$

3. a) solve:  $\frac{d^2y}{dx^2} + y = e^x + \sin 3x$

b) solve:  $\frac{d^2y}{dx^2} - 2 \frac{dy}{dx} + y = x^2 e^{3x}$

4. a) solve:  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = x^2$

b) solve:  $\frac{dx}{x^2 - y^2 - z^2} = \frac{dy}{2xy} = \frac{dz}{2xz}$

5) a) solve:

$$(x^3 - 4x) \frac{d^3y}{dx^3} + (9x^2 - 12) \frac{d^2y}{dx^2} + 18x \frac{dy}{dx} + 6y = 0$$

b) Show that the solution of the initial value problem,  $\frac{dy}{dx} = \sqrt{|y|}$ ,  $y(0) = 0$  may <sup>not</sup> be unique although  $f(x)$ ,  $f(x, y) = \sqrt{|y|}$  is continuous

6) a) solve:

$$x^2 d^2y - 2x(1+x) \frac{dy}{dx} + 2(1+x)y = x^3$$

b) solve:  $\frac{d}{dx} (\cos^2 x \frac{dy}{dx}) + y \cos^2 x = 0$

7) a) solve  $p + 3q = 5z + \tan(y - 3x)$

b) solve the following equation by Charpit's method.

8 a) solve  $y + 5 - 6x = y \cos x$

b) solve -  $\frac{\partial^2 z}{\partial x^2} - 4 \frac{\partial^2 z}{\partial x \partial y} + 4 \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - 2 \frac{\partial z}{\partial y} = e^{x+y}$

9 a) solve:  $\frac{d^2 y}{dx^2} + \cot x \frac{dy}{dx} + 4y \operatorname{cosec}^2 x = 0$

b) solve by the method of variation of parameters

$$(1-x) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = (1-x)^2$$

10) a) solve:  $p \cos(x+y) + q \sin(x+y) = z$

b) solve:  $z(p^2 - q^2) = x - y$

11) a) solve:  $x^3 \frac{d^2 y}{dx^2} - 2x^2 \frac{dy}{dx} + 2xy = 1$

b) solve:  $\frac{dx}{y} = \frac{dy}{x} = \frac{dz}{xyz^2(x^2 - y^2)}$

12) a)  $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$ . solve

b) solve:  $\frac{dy}{dx} = \frac{6x - 2y - 7}{2x + 3y - 6}$

13) a) solve:  $x^2(y - px) = yp^2$

b) solve: discuss the solution of the following equation  $p^2 x^3 + pyx^2 + a^3 = 0$

14) a) solve:  $(D^3 + D^2 - D - 1)y = \cos 2x$

b) solve:  $(D^2 - 4D + 4)y = 8x^2 e^{2x} \sin 2x$

15) a) solve:  $\frac{dy}{dx} = x^3 y^3 - xy$

b) solve:  $(x^2 y - 2xy^2) dx - (x^3 - 3x^2 y) dy = 0$

16) a) solve:  $(D^3 - 3D^2 + 4D - 2)y = e^x + \cos x$

b) solve:  $\frac{d^2 y}{dx^2} + \frac{dy}{dx} = x \cos x$