Department of Mathematics
Semester - II

paper - Calculus

Part-A

1. State Taylor's theorem with Langrang's form of Remainder

2. What is power series, write expression.

3. Write defination of Maclaurin series.

4. Find the Lagrang's foom of Remainder in the ~~expression~~ expansion of the function: a^x

5. Write cartesian formula for derivative of length of arc

6. Find ds/dx for the curve
$$y = c\cosh\left(\frac{x}{c}\right)$$

7. Find ds/dt for the curve
$$x = a\cos^3 t,\ y = b\sin^3 t$$

Q.8 For any curve prove that

$$\sin^2\phi \frac{d\phi}{d\theta} + r\frac{d^2r}{ds^2} = 0$$

Q.9 Find radius of curvature of the point (s, ψ) on the following curves: $s = c\log\sec\psi$

Q.10 Write formula for chord of curvature perpendicular to radii vector.

Q.11 Varify $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ for $u = x^y$

Q.12 Write Euler's theorem for homogeneous function

Q.13 Varify Euler's theorem for

$$u = x^n \sin\left(\frac{y}{x}\right)$$

Q.14 Find the value of $\frac{du}{dt}$, if

$u = x^4 y^5 \quad x = t^2, \; y = t^3$

Q.15 If $y^x + x^y = c$ then find the value of dY/dx

Q.16 Write defination of Envelope

Q.17 Find the envelope of the lines

$$y = mx - 2am - am^3$$

Q.18 Find the maxima and minima of the following function

$$x^2(1-x)^3$$

Q.19 Determine the point where where the function $u = x^2 + y^2 + 6x + 12$ has maxima & minima

Q.20 Find the asymptotes of the following curve

$$\frac{a^2}{x} - \frac{b^2}{y^2} = 1$$

Q.21 Write Types of double points.

Q.22 Write formula for beta function.

Q.23 Evaluate: $\int_0^{\pi/2} \sin^6 x \, dx$

Q.24 Show that

$$\int_0^\infty \sin(x^2)\, dx = \frac{1}{2}\sqrt{\pi/2}$$

Q.25 Evaluate

$$\int_0^3 \int_1^2 xy(1+x+y)\, dy\, dx.$$

Q.26 What is dirichlet's Integral

Q.27 Write Liouville's Extension of dirichlet's integral.

Q.28 Evaluate $\iiint e^{x+y+z}\, dx\, dy\, dz$ taken over positive octant such that $x+y+z \leq 1$

Q.29 Show that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$

Scanned with OKEN Scanner

30. Prove that length of arc of the curve $y = \log \sec x$ from $x=0$ to $x=\frac{1}{3}\pi$ is $\log_e(2+\sqrt{3})$

31 Write the formula for length of the curve in cartesian form and in parametric form.

32. Evaluate

$$\int_0^1 \int_0^1 \frac{dx\,dy}{\sqrt{(1-x^2)(1-y^2)}}$$

33 Show that

$$\int_0^\infty \sin(x^2)\,dx = \frac{1}{2}\sqrt{\pi/2}$$

34 Evaluate $\dfrac{\Gamma 1/3\, \Gamma 5/6}{\Gamma 2/3}$

35. Write Gamma duplication formula

36 prove

$$B(m,n) = B(m+1,n) + B(m,n+1)$$

37 Write relation between Beta and Gamma functions

38. Determine the existence and nature of the double point of the curve

$$(x-2)^2 = y(y-1)^2$$

39. Show that the point of inflexion of the curve $y^2 = (x-a)^2(x-b)$ lie on the line $3x+a=4b$

40. Find the asymptote of the following curve

$$x^3 - y^3 - 3axy = 0$$

Part - B

Q.1 Write Power series Expansion of

$f(x) = \cos x$

Q.2 Find the Maclaurin's series for the

$\tan^{-1} x$

Q.3 Find the pedal Equation for the following curve $y^2 = 4a(x+a)$

Q.4 prove that for any curve $p = f(r)$

$$\frac{ds}{dr} = \frac{r}{\sqrt{r^2 - p^2}}$$

Q.5 Find the radius of curvature at any point 't' of the following curves:

$x = a(t + \sin t)$, $y = a(1 - \cos t)$

Q.6 For the curve $s = m(\sec^3 \psi - 1)$

prove that $\rho = 3m \tan \psi \sec^3 \psi$

and show that

$$3m \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} = 1$$

Q.7 Show that for the curve $s = c e^{x/c}$

$c\rho = s\sqrt{s^2 - c^2}$

Q.8 If $u = \log r$ where $r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$ then prove that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{r^2}$$

Q.9 If $\theta = t^n e^{-r^2/4t}$ then find the value of n for which

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right) = \frac{\partial \theta}{\partial t}$$

Q.10 If $u = f(r)$ where $r^2 = x^2 + y^2$, then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r} f'(r)$.

Q.11. If $x^3 + y^3 = 3axy$ then find $\frac{d^2y}{dx^2}$

Q.12 Find the envelope of the family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$, where two parameter a, b are connected by a relation $ab = c^2$, c being a constant.

Q.13 Find the envelope of the family of straight lines

$$\frac{ax}{\cos\alpha} - \frac{by}{\sin\alpha} = a^2 - b^2$$

where α is the parameter

Q.14 Find the envelope of the family of ellips $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a+b=c$, c is a constant.

Q.15 Show that the minimum value of the following function is $3a^2$

$$u = xy + a^3\left(\frac{1}{x} + \frac{1}{y}\right)$$

Q.16. If $u = a^2x^2 + b^2y^2 + c^2z^2$, find max and min values of u, s.t $x^2+y^2+z^2 = 1$ and $lx + my + nz = 0$

Q.17 Explain kind of cusp

Q.17 Show that the points of intersection of the curve $2y^3 - 2x^2y - 4xy^2 + 4x^3 - 14xy + 6y^2 + 4x^2 + 6y + 1 = 0$ and its asymptote lie on the straight line $8x + 2y + 1 = 0$

Q.18 Determine the double points and their nature on the curve

$$x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0$$

Q.19 Determine the double point and their nature on the curve $y(y-6) = x^2(x-2)^3 - 9$.

Q.20 write kind of cusp

Q.21 Write any two properties of Gamma function

Q.22 $\Gamma n \, \Gamma 1-n = \dfrac{\pi}{\sin n\pi}$, $0 < n < 1$

Q.23 prove $\Gamma n \, \Gamma n+\frac{1}{2} = \dfrac{\sqrt{\pi}}{2^{2n-1}} \cdot \Gamma 2n$ $n \in Z$

Q.24. Evaluate $\int_0^{\infty} x^{m-1} e^{-ax} \cos bx \, dx$.

Q.25 Show that

$$\int_0^{\infty} \sin(x^2)\, dx = \frac{1}{2}\sqrt{\pi/2}$$

Q.26 Evaluate $\int_0^{\cancel{a\sqrt{a^2-y^2}}} \int$

$$\int_0^{a} \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2-x^2-y^2}\, dx\, dy$$

Q.27 Change the order of integration in the following integral

$$\int_0^{a} \int_0^{\sqrt{a^2-x^2}} f(x,y)\, dx\, dy$$

Q.28 Evaluate $\iiint e^{x+y+z}\, dx\, dy\, dz$ taken over positive octant such that $x+y+z \leq 1$

Q.29 Find the area between the of parabola $x^2 = 4y$ and the line $x = 4y - 2$

Q.30 Find the volume in the first octant bounded by the cylinder $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$.

Part - C

Q. 1 Prove that the function defined on R

$$f(x) = \begin{cases} e^{1/x^2}, & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Q. 2 Prove that the number θ which occurs in the taylor theorem with langrange's form of remainder after n terms approaches the limit $\frac{1}{n+1}$ as $h \to 0$, provided $f^{n+1}(x)$ is continous at a and different from zero.

Q. 3 If $\frac{2a}{r} = 1 + \cos\theta$, show that with usual notations ~~to~~ then

$$\frac{ds}{d\psi} = \frac{2a}{\sin^3\psi}$$

Q. 4 Find the pedal Equation for $x^{2/3} + y^{2/3} = a^{2/3}$ (The astroid)

Q.5 Prove that for the ellips $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$:

$\rho=\frac{a^2b^2}{p^3}$, where p is the length of the perpendicular from the centre upon the tangent at (x,y).

Q.6 Show that the radius of curvature at any point (x,y) of astroid $x^{2/3}+y^{2/3}=a^{2/3}$ is three times perpendicular length dropped from the origin on the tangent.

Q.7 If $\frac{x^2}{a^2+u}+\frac{y^2}{b^2+u}+\frac{z^2}{c^2+u}=1$

then prove that

$$\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2=2\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}\right)$$

Q.8 Find the envelope of the family of curve $\sqrt{x/a}+\sqrt{y/b}=1$ when $a^n+b^n=c^n$

Q.9 Find the maxima and minima of
$\sin x + \sin y + \sin(x+y)$

Q.10 Find the maximum and minimum value of $u = x^2+y^2+z^2$ subject to the following condition $ax+by+cz=p$

Q.11 Show that the asymptote of the cubic curve cut the curve again in three points which lie on the straight line ~~x+~~ $x-y+1=0$

~~Q.12 Show that the asymptotes of the cubic cut the~~

Q.12 Find the equation of the hyparabola whose asymptote are $x+y=1$, $x-y=1$ and which passes through the origin

Q.13 trace the curve
$a^2y^2 = x^2(a^2-x^2)$

Q.14 trace the curve

$$y^2(a^2+x^2) = x^2(a^2-x^2)$$

Q.15 trace the curve

$$ay^2 = x(x-a)^2$$

Q.16 trace the curve

$$r = a + b\cos\theta \quad a<b$$

Q.17 prove $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx$

Q.18 Evaluate $\iint_R (x+y)^2\,dx\,dy$

where the area is bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Q.19 Evaluate the following integral by changing the to polar co-ordinates

$$\int_0^1 \int_x^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}\, dx\, dy$$

Q.20 Evaluate

$$\iiint_V 2z\, dx\, dy\, dz$$

where region of integration V is a cone closed by following surfaces

$$x^2+y^2=z^2, \quad z=1$$