A1

Department of Mathematics
Semester I

Paper. Discrete mathematics and optimization techniques.

Part A

Q.1 Discuss the properties of equivalence classes

Q.2 Explain the partial order relations with Example

Q.3 Define the following terms

a) Universal Relation

b) Empty Relation

c) Inverse Relation

d) Reflexive Relation

Q.4 what do you mean by partial order set? Give an example.

Q.5 what are the different properties of lattices?

Q.6 Discuss why lattices are considered as partial ordered set.

Q.7 make the truth table for the statement formula $p \vee \neg q$

Q.8 Let us consider the two function a and b where

$$a_r = \begin{cases} 0 & 0 \le r \le 2 \\ 2^{-r}+5 & r \ge 3 \end{cases}$$

$$b_r = \begin{cases} 3-2^r & 0 \le r \le 1 \\ r+2 & r \ge 2 \end{cases}$$

find
i) $a_r + b_r$
ii) $a_r \cdot b_r$

Q.9 Define Numeric functions with suitable Example

Q.10 Explain Scaling of numeric function

Q.11 what do you mean by convolution of numeric function

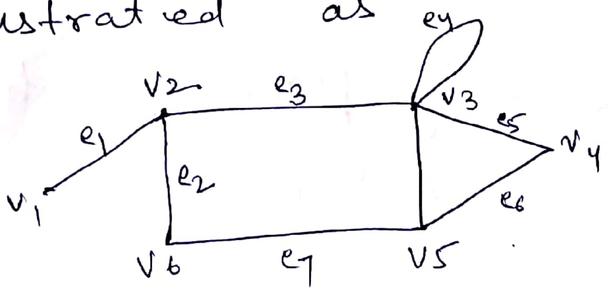
12 Determine the generating function of the sequence A2

$$a_r = \begin{cases} 2^2 & \text{if } r \text{ is even} \\ -2^r & \text{if } r \text{ is odd} \end{cases}$$

Q.13 what is the recurrence relation for the binomial coefficient $C^n(n,k)$

Q.14 Give the solution of recurrence relation by the method of generating function.

Q.15 Find the degree of the vertices as illustrated as



Q.16 prove that a graph is bipartite if all its circuits are of even length.

Q.17 Prove that, the vertices of odd degree in a graph are always even.

Scanned with OKEN Scanner

Q.18 Draw graph representating problems of four houses and four utilites say. gas, water, electricity and telephone.

Q.19 prove that the graph K_5 is non-planar

Q.20 Let G is a graph such that $G=[V,E]$ the number of vertices in graph G is 11. Prove that the graph G or the G complement $(\overline{G})$ is non plannar.

Q.21. what do you mean by planar graph. Explain with proper Example

22. Distinguish between eulerin graph and hamiltonian graph

Q.23. State ~~and~~ ~~prove~~ Dirac's theorem.

Q.24 Simplify the expression

$AB + A(B+C) + B(B+C)$.

Q.25 simplify $\overline{\overline{\overline{ABCD}}}$ using De morgan's Law

Q.25 Define the following terms

1) Boolean function
11) Boolean variables
111) Boolean constant

Q.26 what do you mean by normal form

Q.27 what do you mean by contradiction.

Q.28 Discus the various type of proposition.

Q.29 prove that $(\neg(p \vee q)) \vee (\neg p \wedge q) = \neg p$

Q.30 Write contra positive Rule

Q.31 If $S = \{(x,y) : y^2 \geq 4x\}$, prove that set S is not a convex set

Q.32 prove a hyperplane is convex set

Q.33 Intersection of two convex set is a convex set

Q.34 what is infeasible solution

Q.35 Describe the limitation of of L.P.P.

Q.36 what are the advantages of north-west corner method.

Q.37 what is an unbalanced case in a transportation model

Q.38 what are the advantages of assignment problem

Q.39 what are the limitations of assignment problem

Q.40 Explain Hungarian method of assignment problem.

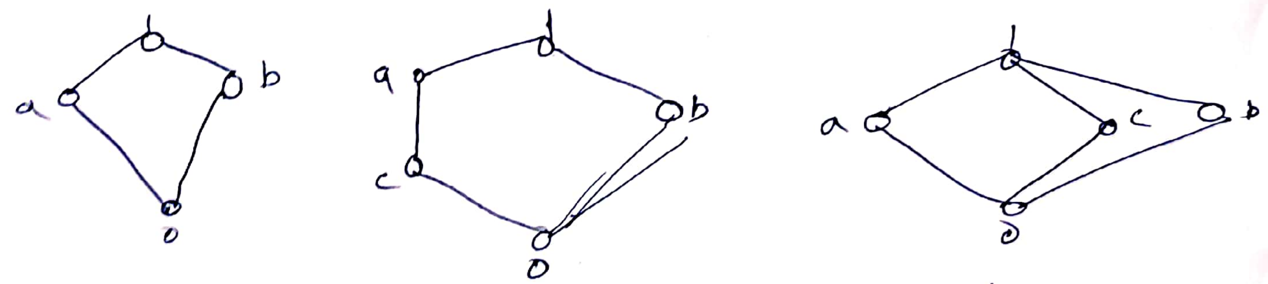
Part B

Q. 1 Show that the Relation R in a set Z of integers given by $R = \{(a,b) : 2 \text{ divides } a-b\}$ is an equivalence Relation

Q. 2. Let A, B, C, D be sets, R_1 is a relation from A to B, R_2 is a relation from B to C and R_3 is a relation from C to D, then prove that $(R_1 \circ R_2) \circ R_3 = R_1 \circ (R_2 \circ R_3)$

Q. 3 Suppose that L is bounded distributive lattice and if compliment of $a \in L$ exist then prove that it is unique.

Q. 4 Show that the lattices shown in figure below are complemented lattices



(a) (b) (c)

Q.5 Write an equivalent formula for

$p \wedge (q \leftrightarrow r) \vee (r \leftrightarrow p)$ which does not involve biconditional.

Q.6 If a, b, c are numeric functions such that $c = ab$, then prove that

$$(\Delta c)(r) = a_{r+1} \cdot (\Delta b)(r) + b_r \cdot (\Delta a))(r)$$

Q.7. Show that every bounded numeric function is $O(1)$.

Q.8. If generating function $f(x) = \frac{1}{(1-ax)^k}$

then determine the discrete numeric

Q.9 Determine the numeric function whose generating function is

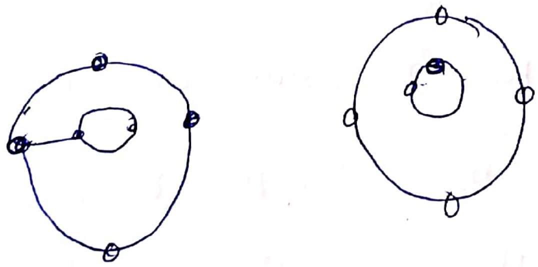
$$\frac{1}{(1-x)^2}$$

B 2

10 Explain ~~et~~ euler graph with example?
prove that a connected graph is eulerian if and only if every vertex has even degree.

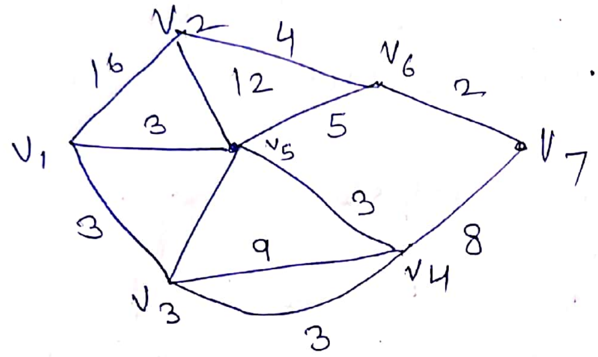
Q.11. Given a graph G with 4 vertices v_1, v_2, v_3 and v_4, where the degrees of the vertices are 3, 5, 2 and 1 resp. Is it possible to construct such a graph G? If not, Explain why?

Q.12 Show that following graph are not isomorphic



Q.13 If G is a simple graph with n vertices and k components prove that G has at-least $n-k$ number of edges.

Q.14 Find the shortest path between vertex V_1 and V_7 with help of dijkstra's algorithm for the figure

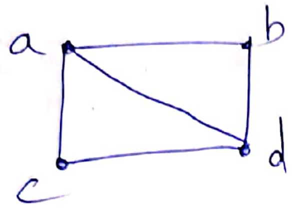


Q.15 prove that two graphs are isomorphic if and only if their complements are isomorphic.

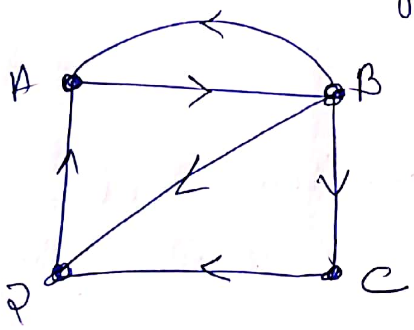
Q.16 prove that G has a Hamiltonian path ~~as~~ tras circuit if $m \geq \frac{1}{2}(n^2 - 3n + 6)$ (where m is the number of edges) n is the number of vertices and G has no loops or multiple edges $n > 2$

Q.17 If G is a graph of order $n \geq 3$, such that $\deg v \geq \frac{n}{2}$ for each vertex v of G, then G is Hamiltonian

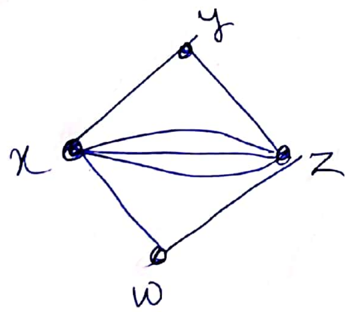
Q.18 Find the adjacency matrix of the following



Q.19 Find the in-degree and out-degree of each vertex of G



Q.20 Find the order size of the graph



Q.21 A connected graph is eulerian if and only if every vertex has even degree.

22 Let G be a connected multigraph.

then

23. If a and b be two numeric function

given by

$a_r = r + o\left(\frac{1}{r}\right)$. $b_r = \sqrt{r} + o\left(\frac{1}{\sqrt{r}}\right)$

prove that $c_r = a_r b_r = r^{3/2} + o(\sqrt{r})$

Q.24 Solve the following LPP

max $Z = 2x_1 + 4x_2$

s.t.

$2x_1 + x_2 \leq 18$

$3x_1 + 2x_2 \geq 30$

$x_1 + 2x_2 = 26$, $x_1, x_2 \geq 0$

Q.25 Solve by Big M Method.

Minimize $Z = 60x_1 + 80x_2$

s.t.

$x_2 \geq 200$

$x_1 \leq 400$

$x_1 + x_2 = 500$, $x_1, x_2 \geq 0$

Q.26 For the given Transportation problem obtain Initial BFS by VAM

						Availability
9	12	9	6	9	10	5
7	3	7	7	5	5	6
6	5	9	11	3	11	2
6	8	11	2	2	10	9
4	4	6	2	4	2	

Demand (last row)

Q.27 Consider the following table that shows manufacturing plants and warehouses of company and the shipping cost to transportation the vailable quantity from plant A, B, C to demand at every warehouses X, Y, Z and W

Plant	X	Y	Z	W	supply
A	3	2	5	2	15
B	2	1	4	4	24
C	2	3	4	3	21
Demand	13	12	16	19	

Q.28 Find the initial basic feasible of TP by NWC method

	P	Q	R	Supply
A	4	9	6	7
B	5	5	3	10
C	7	6	9	9
D	3	8	4	16
	9	13	20	42

Q.29 Solve the following TP using least cost method

	A_1	A_2	A_3	A_4	
X_1	8	12	8	6	4500
X_2	7	15	7	13	6000
X_3	5	9	10	11	7000
Demand	6000	4500	3000	4000	

Q.30 Solve the following AP using Hungarian method

Q.

Men	Tasks A	B	C	D
1	45	40	50	67
2	57	42	63	55
3	49	51	48	64
4	41	45	60	55

Part-C

Q.1 A lattice L is distributive if and only if $(a \vee b) \wedge (b \vee c) \wedge (c \vee a) = (a \wedge b) \vee (b \wedge c) \vee (c \wedge a) \ \forall \ a, b, c \in L$

Q.2 Determine the CNF of given statements

1) $(p \wedge \neg(q \wedge r)) \vee (p \rightarrow q)$
II) $(q \vee (p \wedge q)) \wedge \neg((p \vee r) \wedge q)$
III) $(p \wedge \neg(q \vee r)) \vee (((p \wedge q) \vee \neg r) \vee p)$

Q.3 Show that the following statements are logically equivalent without using truth table

1) $(P \rightarrow Q) \wedge (P \rightarrow R) \Leftrightarrow P \rightarrow (Q \wedge R)$
II) $\neg(P \vee (\neg P \wedge Q)) \Leftrightarrow \neg P \wedge \neg Q)$

Q.4 Suppose $a_r = \begin{cases} 0 & 0 \le r \le 4 \\ r^2 + 1 & r \ge 5 \end{cases}$

determine Δa_r and ∇a_r

Q.5 A person deposits R 200 in a saving account at an interest rate 6% per year compounded annually obtained the amount in this accont at the end of r year find the numeric function

Q.6 Solve the recurrence relation

$$a_{n+2} - 8a_{n+1} + 16a_n = n4^n$$

Q.7 Determine the perticular solution of the given difference equation:

$$a_r + a_{r-1} = 2r \cdot 3^r \quad r \geq 1$$

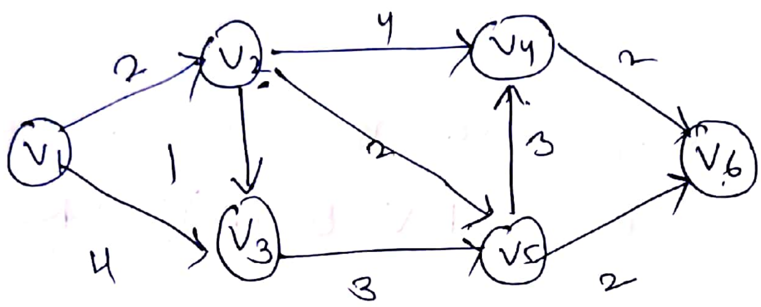
Q.8 Find the solution of recurrence relation

$7a_{r-1} + 10a_{r-2} = 0$ for $n \geq 2$ given that

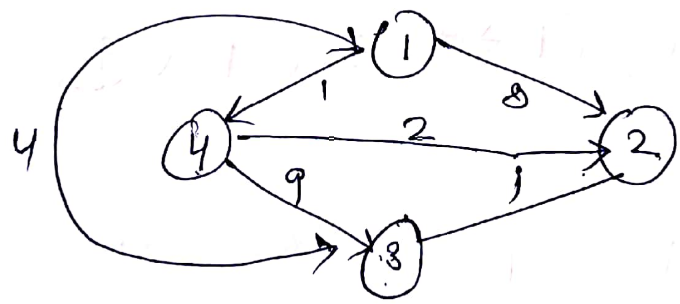
$a_0 = 10, a_1 = 41$ using generating functions.

C2

Q.9. Apply Dijkstra's algorithm to find the shortest path between v_1 and v_6



Q.10 Let us consider the given directed weighted graph



Now determine the shortest path distence between every pair of p. vertices by using floyd warshall Algorithm.

11) Prove the following De Morgan's law

1) $(a.b)' = a' + b'$

11) $(a+b)' = a'.b' \ \forall \ a, b \in B$

11) Obtain the disjunctive and conjuctive normal forms of $(P \vee (Q \wedge R)) \wedge R$

12) Obtain the PCNF of the formula S given by $(\neg P \rightarrow R) \wedge (Q \Leftrightarrow P)$

13) Prove that $(P \rightarrow q) \equiv (\bar{P} \vee q)$

14) Determine the perticular Integral solution of the given difference Equation

$$a_r + 3a_{r-1} = 2r . 3^r \qquad r \geq 1$$

15) Show that in a simple graph with n vertices the max number of edges is $\frac{n(n-1)}{2}$ and the maximum degree of any vertex is $n-1$

16 Use simplex method to solve LPP

min $z = x_1 - 3x_2 + 2x_3$

s.t.

$3x_1 - x_2 + 2x_3 \le 7$

$-2x_1 + 4x_2 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$, $x_1, x_2, x_3 \geq 0$

17 Use simplex method to solve LPP

max $z = x_1 + x_2 + 3x_3$

s.t.

$3x_1 + 2x_2 + x_3 \le 3$

$2x_1 + x_2 + 2x_3 \le 2$, $x_1, x_2, x_3 \geq 0$

18. Solve TP using VAM.

from/to	x	y	z	Demand
P	18	12	3	150
Q	9	24	21	120
R	12	12	6	180
Supply.	60	285	105	

19 Solve the following AP

	I	II	III	IV	V
1	11	17	8	16	20
2	9	7	12	6	15
3	13	16	15	12	16
4	21	24	17	28	26
5	14	10	12	11	15

20 Consider the problem of assigning five Jobs to five person. the assignment cost are given as following.

Person \ Jobs	1	2	3	4	5
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Determine the optimum assignment ~~cost~~ schedule.