B.Sc. Part-I : Mathematics

Paper-II — Calculus

1. a) Test the convergence of the following series:

$$\frac{2x}{1^2}+\frac{3^2x^2}{2^3}+\frac{4^3x^3}{3^4}+\frac{5^4x^4}{4^5}+$$

b) Test the convergence and absolute convergence of the following series:

$$1-\frac{1}{2^p}+\frac{1}{3^p}-\frac{1}{4^p}+$$

2. (a) Prove that the hyper-harmonic series will be convergent if $p>1$ and divergent if $p\leq 1$, where

$$\Sigma\frac{1}{n^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\quad +\frac{1}{n^p}+$$

(b) Examine the convergence of the series:

$$\frac{\sqrt{2}-\sqrt{1}}{1}+\frac{\sqrt{3}-\sqrt{2}}{2}+\frac{\sqrt{4}-\sqrt{3}}{3}+$$

3. (a) Find the pedal equation of the parabola $y^2=4a(x+a)$

(b) If the centre of curvature of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ of one end of the minor axis at the other end. Show the eccentricity of the ellipse is $\frac{1}{\sqrt{2}}$.

4) a) If $x^x y^y z^z = c$, prove that at $x=y=z$

$$\frac{\partial^2 z}{\partial x \partial y} = -(x \log ex)^{-1}$$

b) If $u = \tan^{-1} \frac{x^3+y^3}{x+y}$, then prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \sin 2u(1-4\sin^2 u)$$

5) a) find the envelope of the straight lines $x \cos\alpha + y \sin\alpha = l \sin\alpha \cos\alpha$. a being the parameters.

b) find the maxima and minima of $u = x^2+y^2+z^2$ subjects to the conditions $ax^2+by^2+cz^2=1$ and $lx+my+nz=0$

6) a) find the asymptotes of the following curve:

$$y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 - 1 = 0$$

b) Trace the following curve :- $y = \frac{x^2+1}{x^2-1}$

7) a) Show that: $\int_0^\infty \sin(x^2)\,dx = \frac{1}{2}\sqrt{\pi/2}$

b) evalute the following integral by changing to polar coordinates:

$$\int_0^a \int_y^a \frac{x\,dy\,dx}{x^2+y^2}$$

8) a) change the order of integration in the double integral

$$\int_0^{2a}\int_{\sqrt{(2ax-x^2)}}^{\sqrt{(2ax)}} f(x,y)\,dx\,dy$$

b) Evaluate : $\iiint_v x^2\,dx\,dy\,dz$

where the region v is enclosed by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=a$

9) a) find the area included between the curve $xy^2=4a^2(2a-x)$ and its asymptote.

b) Test the convergence of the series

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\infty$$

0) a) Test for convergence the series whose nth term is $n^{\log(x)}$

b) Test the following series for convergence:

$$\frac{x^2}{2\log 2}+\frac{x^3}{3\log 3}+\frac{x^4}{4\log 4}+$$

11. a) use Taylor's Theorem to prove that:

$$\tan^{-1}(x+h) = \tan^{-1}x + h\sin z \cdot \frac{\sin(z)}{1} - (h\sin z)^2 \frac{\sin(2z)}{2} + h(\sin z)^3 \frac{\sin(3z)}{3}$$

where $z = \cot^{-1}(x)$

b) If ρ_1 and ρ_2 be the radii of curvature at the extremities of two conjugate diameters on an ellipse, prove that:

12. a) find the whole length of the astroid:

$$x^{2/3} + y^{2/3} = a^{2/3}$$

b) find the pedal equation of the parabola. $y^2 = 4a(x+a$

13. a) If $x^x y^y z^z = \lambda$ prove that at $x = y = z$

$$\frac{\partial^2 z}{\partial x \partial y} = -(x \log_e x)^{-1}$$

b) Trace the following curve: $ay^2 = x^2(a-x)$

14) a) find the asymptotes of the following curve:

$$y^3 - x^2y + 2y^2 + 4y + x = 0$$

b) prove that:

$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$$

15. a) find the envelope of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters a and b are related by equation $a^n + b^n = c^n$, c being a constant.

b) find the maximum or minimum value of $u = a^2x^2 + b^2y^2 + c^2z^2$ subject to the conditions $x^2 + y^2 + z^2 = 1$ and $lx + my + nz = 0$

16. a) Evaluate $\iint (x^2+y^2)\,dx\,dy$ over the region in the positive quadrant for which $x + y \le 1$

b) change the order of integration and evaluate

$$\int_0^{\infty}\int_{e^x}^{a} \frac{dx\,dy}{\log y}$$

17 a) Evaluate $\iiint_V zy^2\,dx\,dy\,dz$, where V is the region bounded between x-y plane and the sphere $x^2+y^2+z^2=1$

b) find the area common between the curves $y^2 = 4ax$ and $x^2 = 4ay$

18. a) find the intrinsic equation of $y^2 = ax^2$.

b) find the volume of solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about x axis.

19. a) find the area of the surface of revolution of the loop of the curve $9ay^2 = x(3a-x)^2$ about x-axis.