B.Sc. Part- I

Mathematics- III

Analytic geometry and optimization Techniques

Q.1. A sphere of constant radius r passes through the origin O and cuts the axes in A, B, C. Prove that locus of feet of perpendicular drawn from O to the plane ABC is given by

$$(x^2+y^2+z^2)^2(x^{-2}+y^{-2}+z^{-2}) = 4r^2$$

Q.2. Find eq^n of sphere described on the line joing points $(2,-1,4)$ & $(-2,2,-2)$ as diameter. Also find the area of circle in which this sphere cut by plane $2x+y-z=3$

Q.3 Prove that the centres of spheres which touch the lines $y=mx, z=c$ and $y=-mx, z=-c$ lie upon the conecoide $mxy+cz(1+m^2)=0$

Q.4. Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of common circle is $r_1 r_2/\sqrt{r_1^2+r_2^2}$

Q.5 Show that eq^n of cone whose vertex is the origin and guiding curve $z=K, f(x,y)=0$ is $f\left(\frac{xK}{z}, \frac{yK}{z}\right)=0$

Q6. Prove that angle between lines in which plane $x+y+z=0$ cuts a cone $ayz+bzx+cxy=0$ is $\pi/3$ if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$

Q.7 Prove that the semi vertical angle of a right circular cone admitting sets of three mutually perpendicular generators is $\tan^{-1}\sqrt{2}$

Q8 Find eqⁿ of cylinder whose generators are parallel to $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is ellipse $x^2 + 2y^2 = 1, z = 0$.

Q.9. Find eqⁿ of a right circular cylinder whose guiding circle passes through $(a,0,0)$, $(0,b,0)$, and $(0,0,c)$.

Q.10. A tangent plane to ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c} = 1$ meets the co-ordinate axes in points A, B, C. Prove that the centroid of the triangle ABC lies on the locus

$$\frac{a^2}{x^2} + \frac{b^2}{y^2} + \frac{c^2}{z^2} = 9$$

Q.11. Prove that the feet of six normals from (α, β, γ) to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ lie on the curve of intersection of the ellipsoid and cone

$$a^2(b^2-c^2)\frac{\alpha}{x} + \frac{b^2(c^2-a^2)\beta}{y} + \frac{c^2(a^2-b^2)\gamma}{z} = 0$$

Q.12. find eqⁿ of generating lines of hyperboloid $yz + 2zx + 3xy + 6 = 0$ which passes through the point $(-1, 0, 3)$

Q.13. Show that the plane through two intersecting generators is the tangent plane of hyperboloid of one sheet at their common point

Q.14. Find principle directions and principle planes of the following conicoids:-

$$2x^2+20y^2+18z^2-12yz+12xy+22x+6y-8z-8=0$$

Q.15. Reduce following eq^n in Canonical form and find the nature of surface represented by it:- $4x^2+9y^2+36z^2-36yz+24zx-12xy$

$$-10x+15y-30z+6=0$$

Q.16. PSP' is focal Chord of conic, prove that tangent at P & P' intersect on the directrix.

Q.17. Find the condition that straight line $\frac{l}{r} = A\cos\theta + B\sin\theta$ may touch circle

$$r = 2a\cos\theta.$$

Q.18. If normal at α, β, γ on $\frac{l}{r} = 1+e\cos\theta$ meet in the point $P(\rho, \varphi)$ show that

$$2\varphi = \alpha+\beta+\gamma$$

Q.19. Prove that two conic $\frac{l_1}{r} = 1+e_1\cos\theta$ & $\frac{l_2}{r} = 1+e_2\cos(\theta-\alpha)$ will touch each other if

$$l_1^2(1-e_2^2) + l_2^2(1-e_1^2) = 2l_1 l_2(1-e_1e_2\cos\alpha$$

Q.20. Prove that if primal problem has unbounded solution then dual has either no solution or unbounded Solution.

a L.P.P. is a convex set

Q.23. Show that the feasible solⁿ of following system of eqⁿ $x_1 = 1, x_2 = 0, x_3 = 1, z = 6$ is not basic

$$x_1 + x_2 + x_3 = 2$$
$$x_1 - x_2 + x_3 = 2$$
$$x_i \geq 0 \quad i = 1, 2, 3$$

Q.24. Show that following set is convex set

$$S = \{x : x = (x_1, x_2, x_3);\ x_1^2 + x_2^2 + x_3^2 \leq 1\}$$

Q.25. Write the dual problem of given L.PP

$$\text{Min } Z = x_1 - 3x_2 - 2x_3$$

s.t.

$$3x_1 - x_2 + 2x_3 \leq 7$$
$$2x_1 - 4x_2 \geq 12$$
$$-4x_1 + 3x_2 + 8x_3 = 10$$

$x_1, x_2 \geq 0$ & x_3 is unrestricted in sign.