

Kanoria PG Mahila Mahavidyalaya, Jaipur

Department of Computer Science

202: Discrete Mathematics

BCA II

Question Bank

1. Let A be any finite set and $P(A)$ be the power set of A . \subseteq be the inclusion relation on the elements of $P(A)$. Draw the Hasse diagrams of $(P(A), \subseteq)$ for i) $A = \{a\}$ ii) $A = \{a,b\}$ iii) $A = \{a,b,c\}$ iv) $A = \{a,b,c,d\}$.
2. Let $A = B = \{x / -1 \leq x \leq 1\}$ for each of the following functions state whether it is injective, surjective or bijective
 - a) $f(x) = |x|$
 - b) $g(x) = \sin \pi x$
 - c) $h(x) = 2x+3$
3. Show that the relation $R = \{(a,a), (a,b), (b,a), (b,b), (c,c)\}$ on $A = \{a,b,c\}$ is an equivalence relation and find A/R also find partitions of A .
4. Let $f: R \rightarrow R$, $g: R \rightarrow R$, where R is the set of real numbers be given by $f(x) = x^2 - 2$ and $g(x) = x+4$ find fog and gof. State whether these functions are bijective or not.
5. Prove that the relation R defined by "a is congruent to b modulo m" on the set of integers is an equivalence relation.
6. Define the following : (a) recursive function (b) Total function (c) Partial function.
7. Draw the Hasse diagram representing the positive divisors of 45.
8. If R denotes a relation on the set of all ordered pairs of positive integers by $(a,b)R(c,d)$ iff $ad=bc$, show that 'R' is an equivalence relation.
9. Let $X = \{1,2,3,4,5\}$ and relation $R = \{(x,y) / x > y\}$. Draw the graph of 'R' and also give its matrix.
10. What is Compatibility relation and Write the procedure to find maximal compatibility blocks.
11. Draw the Hasse diagram representing the positive divisors of 36.
12. Show that the relation 'R' defined by $(a,b)R(c,d)$ iff $a+d=b+c$ is an equivalence relation.
13. If $X = \{1,2,3,4\}$ and $R = \{(x,y) / x < y\}$. Draw the graph of 'R' and also give its matrix.
14. If R is a relation on the set $A = \{1,2,3,4\}$ defined by xRy if x exactly divides y . Prove that (A,R) is a poset.
15. Let f and g be functions from R to R defined by $f(x) = ax+b$, $g(x) = 1-x+x^2$. If $(gof) = 9x^2-9x+3$, determine a, b .
16. Let $D_{24} = \{1,2,3,4,6,8,12,24\}$ and the relation "I" exactly divides be a partial ordering relation on D_{24} . Then draw the Hasse Diagram of (D_{24}, I) .
17. Consider the function $f: R$ to R defined by $f(x) = 2x+5$, another function $g(x) = (x-5)/2$. Prove that g is inverse of f .
18. Let $R = \{ [1,1] [2,2] [3,3] [4,4] [5,5] [1,2] [2,1] [5,4] [4,5] \}$ be the equivalence relation on $A = \{1,2,3,4,5\}$. Find equivalence classes and A/R .
19. Find the inverse of the function $f(x) = e^x$ defined from R to R^+ .
20. If $A = \{1,2,3,4,5,6,7,8,9,10,11,12\}$ and $R = \{(x,y) / x-y \text{ is multiple of } 5\}$ find the partition of A .
21. Let $f(x) = x+2$, $g(x) = x-2$, $h(x) = 3x$ find i) fog ii) fogoh.
22. Give an example of relation which is symmetric but neither reflexive nor anti symmetric nor transitive.
23. Use Mathematical Induction to prove the generalization of Demorgan's laws.
24. Prove that if a/bc and $(a,b)=1$ then a/c .
25. State and Prove Division algorithm theorem using well ordering principle.
26. Describe set of rooted trees recursively?
27. Show that if a, b, c are integers such that a/b and a/c then $a/mb+nc$ where m, n are integers.
28. Write the Procedure for Euclidean algorithm to find gcd of two numbers.
29. Describe full binary tree recursively.
30. Prove that there are infinitely many primes.
31. Define modular arithmetic? Prove that the integers a, b are congruent modulo m iff there is an integer k such that $a = b + km$, where m is a positive integer.

32. State fundamental theorem of arithmetic hence find the prime factorization of 810.
33. Write prime numbers less than 150.
34. Write the properties of gcd.
35. State and prove Euclid's lemma.
36. Define Fibonacci numbers recursively.
37. Explain about Mathematical Induction.
38. Explain pair wise relatively primes with an example.
39. Define Mersenne prime numbers.
40. Write the properties of divisibility.
41. Define well ordering principle.
42. Find PCNF without Constructing truth table $(P \rightarrow (Q \wedge R)) \rightarrow (\sim P \rightarrow (\sim Q \wedge \sim R))$
43. Use truth table to prove the following argument
- $$\begin{array}{l}
 p \rightarrow \sim q \\
 r \rightarrow p \\
 q \\
 \therefore \sim r
 \end{array}$$
44. Find PDNF by constructing its PCNF of $(Q \vee P) \wedge (Q \vee R) \wedge (\sim (P \vee R) \vee \sim Q)$
45. Find whether the following argument is valid or not
 "No Engineering student is bad in studies"
 "Anil is not bad in studies"
 Therefore "Anil is an engineering student"
46. Without constructing truth table find PDNF of
 $\{(P \rightarrow (Q \wedge R)) \wedge (\sim Q \wedge \sim R)\}$
47. Prove that the following argument is valid:
 "all dogs are carnivorous."
 "some animals are dogs."
 Therefore "some animals are carnivorous".
48. Is the following Conclusion valid derive from contradiction method.
- $$\begin{array}{l}
 \sim q \\
 p \rightarrow q \\
 p \vee t \\
 \therefore t
 \end{array}$$
49. Construct PCNF of $(P \Leftrightarrow Q) \rightarrow R$.
50. Obtain CNF of $((P \rightarrow Q) \wedge \sim Q) \rightarrow \sim P$
51. Obtain DNF of $(Q \rightarrow P) \wedge (\sim P \wedge Q)$
52. Find PDNF by constructing the PCNF of $(Q \vee P) \wedge (Q \vee R) \wedge (\sim (P \vee R) \vee \sim Q)$.
53. Prove that for any three propositions P,Q,R the compound proposition $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$ is a tautology by i) with truth table ii) with laws of logic.
54. Show that the following set of premises are inconsistent
 $P \rightarrow Q, P \rightarrow R, Q \rightarrow \sim R, P$
55. Check the validity of the following argument
 All integers are rational numbers.
 Some integers are powers of 5.
 Therefore, some rational numbers are powers of 5.
56. Define the following terms. Give one suitable example for each
 (i) Euler circuit (ii) Hamiltonian graph.
57. State and prove Euler's theorem on plane graphs.
58. Define isomorphism of graphs. What are the steps followed in discovering the Isomorphism.
59. Define dual and Isomorphism of graphs with example.
60. Define the following terms with suitable example i) Complete graph ii) Regular graph.
61. Define isomorphism of two graphs.
62. Define the following terms with suitable example i) Subgraph ii) Spanning graph.

63. Define the following sub graphs of a graph a) Closed Walk and Open walk b) Trail
 64. Define Chromatic number of a graph. Explain it through an example.
 65. Draw K_5 complete graph.
 66. State fundamental theorem of graph theory.
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